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AN EMPIRICAL METHOD FOR THE  
DETERMINATION OF GRAVITY TERRAIN CORRECTIONS

A THESIS SUBMITTED TO THE GRADUATE DIVISION OF THE  
UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT  
OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE  
IN GEOLOGY AND GEOPHYSICS

AUGUST 1974

By

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## ABSTRACT

Terrain corrections are usually estimated by laborious determinations of the topographic relations around the gravity station out to distances of 22 to 160 kms on a zone and compartment basis. A systematic measuring of the maximum topographic slope ( $\alpha$ ) and the mean relative relief ( $\Delta h$ ) out to different distances around gravity stations for which the terrain correction was known showed that a set of empirically derived curves could be established relating the terrain correction to the topographic slope and relative relief within approximately 15 kms of the station. Mathematical definition of this set of curves is accomplished through the use of a simplified solid geometrical model (a cone) to define the terrain effect around the gravity station. The equation for this model is used to determine terrain correction values for a sample of 254 gravity stations in the Sierra Nevada Mountain area where the actual terrain corrections as determined by the U. S. Geological Survey using standard rigorous procedures is as high as 74 mgals. A comparison of values shows that the method has an average percentage reliability of about 23 percent, or expressed in terms of the standard deviation of the differences the mgal deviation from the actual terrain corrections for the gravity stations in the test

area was 4.8 mgals. This degree of reliability is verified by further tests against a large sample of 1243 terrain corrected stations established by the U. S. Geological Survey covering a number of physiographic provinces extending from the coast of California into the Klamath Mountains and northern Sierra Nevada Mountains. In this further test, interpolated graphical solutions based on theoretical curves derived for different values of  $\alpha$  and  $\Delta h$  were used.



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## 1.0 Introduction

Bouguer gravity anomalies which are commonly used for both geologic and geodetic studies are referred to as simple or complete. In either case, the anomaly represents the free air gravity anomaly (FA) plus a correction for the gravity effect of the topographic mass above sea level that lies below and around the observation site. This correction in its complete form has three components:

- (1) The slab or plate attraction ( $\Delta g_S$ )

$$\Delta g_S = 2\pi\gamma\sigma h \quad (1)$$

in which  $\gamma$  is the gravitational constant,  $h$  is the elevation of the observation site above sea level, and  $\sigma$  is the mean crustal density which is usually taken to be  $2.67 \text{ gm/cm}^3$ . In metric units using  $\sigma = 2.67$  and expressing  $h$  in meters

$$\Delta g_S = 0.1118 h \text{ mgals} \quad (2)$$

(2) The effect of earth curvature ( $\Delta g_C$ ). This involves a small correction that is a function of elevation.  $\Delta g_C$  reaches a maximum value of  $+1.7 \text{ mgal}$  for an elevation of 2,000 to 2,500 meters above sea level and then decreases to  $-1.0 \text{ mgal}$  for an elevation of 5,000 meters.

(3) The terrain correction ( $\Delta g_T$ ). This correction is always negative in sign relative to the plate correction. The correction allows for the gravity effect of terrain

rising above or falling below the elevation of the station. Under extreme conditions this correction may exceed 50 mgals, but is commonly 15 to 20 mgals in mountainous regions with average topographic relief.

The complete Bouguer correction is thus

$$\Delta g_B = \Delta g_S + \Delta g_C - \Delta g_T \quad (3)$$

The complete Bouguer anomaly is  $(BA) = FA - \Delta g_B$ . In the case of the simple Bouguer anomaly, only the first correction term, the plate effect ( $\Delta g_S$ ), is used, and whereas this simplified form of the Bouguer anomaly does not introduce significant errors in most regions since the terrain correction in most areas does not exceed 3 mgal, in mountainous regions, omission of the terrain correction can result in a significant difference in values.

Despite recognition of the importance of the terrain correction, most of the world's gravity data have only been reduced to give the simple Bouguer anomaly value. This is because of the extensive labor required in making the correction. As commonly done, this involves, even over a limited area of about 25 km radius around a site making estimations of elevation for some 130 odd areas of varying size forming compartments of circular zones around the observation site. These have varying radii and were established on the basis of areas having near equal gravitational effect for a given difference in elevation.

The present study is an attempt to devise a simplified method for determining the terrain correction having sufficient accuracy for most geodetic studies (better than  $\pm 5$  mgal in areas of mountainous relief). On the basis of a test evaluation of the method devised against a representative body of data for the Sierra Nevada Mountain region in California for which complete Bouguer anomaly values were available, it appears the method meets this criterion.

## 2.0 The Zonal System of Terrain Correction

As stated in the Introduction, the effect of the terrain correction is to reduce the gravity effect calculated on the assumption of a slab or plate having the elevation of the gravity observation site. The correction is based on special tables giving the gravitational effect for the difference in elevation of the observation site relative to that of the compartments of circular zones centered on the station and is carried out to such distance that the correction loses significance. The correction itself represents the sum of the various compartmental values for all the zones considered as being significant. The considerable labor involved in this procedure can be gaged from Table 1, defining the zones and compartments used in the Hayford-Bowie (1912) zonal correction system, and from Table 2 which defines those used in the Hammer (1939)

TABLE 1  
HAYFORD-BOWIE ZONES FOR TERRAIN CORRECTION

---

Zone	Compartments	Outer Radius (meters)
A	1	2
B	4	68
C	4	230
D	6	590
E	8	1,280
F	10	2,290
G	12	3,520
H	16	5,240
I	20	8,440
J	16	12,400
K	20	1,880
L	24	28,800
M	14	58,800
N	16	90,000
O	28	16,700 = 1°29'58"

---



TABLE 2  
HAMMER ZONES FOR TERRAIN CORRECTION

Zone	Compartments	Outer Radius	
		Feet	Approx. meters
A	1	6.6	2
B	4	54.6	17
C	6	175	53
D	6	558	170
E	8	1,280	390
F	8	2,936	890
G	12	5,018	1,528
H	12	8,578	2,620
I	12	14,662	4,460
J	16	21,826	6,660
K	16	32,490	9,860
L	16	48,365	14,700
M	16	71,996	29,500

zonal correction system. The only difference between these two systems is that the Hammer correction zones give a finer breakdown of the near station topography and do not consider topography beyond 13.6 miles (22 km) from the observation site whereas the Hayford-Bowie zones consider the surrounding terrain out to 166.7 km (1.50°). The more limited area considered in the Hammer system is because at distances greater than 22 km the effect of the surrounding topography falls off rapidly, and in all but mountainous areas is not significant. This is brought out in Table 3, which reflects a modification by Swick (1942) for easier use of the Hayford-Bowie zone method and in which a density of 2.67 gm/cm<sup>3</sup> is used for the topography. It should be noted that Table 3, while reflecting the effect of earth curvature in that the correction terms in Zone M differ as to whether the terrain lies above or below the elevation of the station, does not embody the earth curvature correction per se which is independently evaluated from Table 4 and as seen is a function of station elevation.

### 3.0 The U. S. Geological Survey Terrain Correction System

As once the regional pattern of elevation values has been established for an area the reduction procedure can be rapid, there have been various attempts made to devise ways of reducing the labor involved in getting the equivalent of

TABLE 3

TERRAIN CORRECTION IN 0.01 MGAL UNITS PER COMPARTMENT HAYFORD-BOWIE ZONES

Elev. Diff. (Ft)	B	C	D	E	F	G	H	I	J	K	L	M	
												Above Sta.	Below Sta.
												Density = 2.67 gm/cm <sup>3</sup>	
10	4	--	--	--	--	--	--	--	--	--	--	--	--
20	11	--	--	--	--	--	--	--	--	--	--	--	--
40	25	1	--	--	--	--	--	--	--	--	--	--	--
60	39	5	1	--	--	--	--	--	--	--	--	--	--
100	61	12	2	1	--	--	--	--	--	--	--	--	--
150	84	27	5	1	1	--	--	--	--	--	--	--	--
200	99	43	9	2	1	--	--	--	--	--	--	--	--
300	121	79	20	5	2	1	--	--	--	--	--	--	--
500	144	148	49	14	4	2	1	1	--	--	--	--	--
800	--	224	106	36	11	4	2	1	1	1	--	--	--

TABLE 3. (Continued) TERRAIN CORRECTION IN 0.01 MGAL UNITS  
PER COMPARTMENT HAYFORD-BOWIE ZONES

Elev. Diff. (Ft)	B	C	D	E	F	G	H	I	J	K	L	M	
												Above Sta.	Below Sta.
												Density = 2.67 gm/cm <sup>3</sup>	
1,200	--	--	182	75	25	10	4	3	2	1	1	--	--
1,600	--	--	250	120	43	17	8	5	3	2	1	3	1
2,000	--	--	304	169	65	26	12	8	5	3	2	4	1
2,500	--	--	358	230	97	39	19	12	8	5	3	6	3
3,000	--	--	400	287	131	54	26	17	11	7	4	8	4
3,500	--	--	--	339	168	73	35	23	15	9	5	10	6
4,500	--	--	--	430	242	113	56	37	25	15	9	16	11
5,500	--	--	--	--	315	158	82	54	37	22	12	23	17

TABLE 3. (Continued) TERRAIN CORRECTION IN 0.01 MGAL UNITS  
PER COMPARTMENT HAYFORD-BOWIE ZONES

Elev. Diff. (Ft)	B	C	D	E	F	G	H	I	J	K	L	M	
												Above Sta.	Below Sta.
												Density = 2.67 gm/cm <sup>3</sup>	
6,500	--	--	--	--	381	204	110	74	51	30	17	31	24
8,000	--	--	--	--	469	274	156	109	76	45	25	47	38
9,000	--	--	--	--	--	318	188	135	95	57	32	58	48
10,000	--	--	--	--	--	360	220	162	115	69	30	71	60
12,000	--	--	--	--	--	--	284	220	162	99	58	101	87
14,000	--	--	--	--	--	--	346	280	213	133	78	136	--
16,000	--	--	--	--	--	--	--	342	269	170	101	175	--

TABLE 4  
EARTH CURVATURE CORRECTION IN 0.1 MGAL  
UNITS AS A FUNCTION OF SURFACE ELEVATION

Sta. Elv. (meters)	Curvature Corr.
0	0
100	+ 2
200	+ 3
300	+ 4
400	+ 6
500	+ 7
600	+ 8
700	+ 9
800	+10
900	+11
1,000	+12
1,500	+15
2,000	+17
2,500	+17
3,000	+15
3,500	+11
4,000	+ 6
4,500	- 1
5,000	-10

a Hammer or Hayford-Bowie zonal representation of topographic changes around a site. One such method used by the U. S. Geological Survey (Oliver, 1965), uses 1 to 20 unit squares centered on the station where the unit square can be 1, 2, 3, 4, or 5 km to approximate the Hammer or Hayford-Bowie zones and compartments out to 20 km. By having mean elevations for each size square on a series of maps the requisite data can be rapidly compiled for automatic computation for all but the central area immediately adjacent to the observation site. The correction for this central core area is based on hand-sketched maps made by the field observer at the time of observation.

#### 4.0 The Vertical Angle Method of Terrain Correction

Another approach is to use the topographic slope angle of the terrain along a number of azimuths surrounding the site as determined in the field and/or from maps. The logic for this approach can be illustrated by using the correction Tables of Hammer (1939) to define what elevation difference is required at different distances to give terrain corrections of 0.01, 0.05, 0.10, and 0.15 mgals along a single azimuth. These data are tabulated in Table 5.

If the data from Table 5 for zones D through H (radial distance 175-8,578 ft.) are plotted in terms of the required elevation difference as a function of mean

TABLE 5

ELEVATION DIFFERENCES REQUIRED USING  
HAMMER TABLES FOR TERRAIN CORRECTIONS OF  
0.1, 0.5, 1.0, and 1.5 MGALS ON A SINGLE AZIMUTH

Correction for a density of 2.0 gm/cm <sup>3</sup>				
Zone	0.01 mgal Elv. Diff. (Ft.)	0.05 mgal Elv. Diff. (Ft.)	0.1 mgal Elv. Diff. (Ft.)	0.1 5 mgal Elv. Diff. (Ft.)
B	4-7	14-16	27-30	--
C	15-24	45-51	74-80	104-110
D	26-43	76-84	114-120	146-152
E	58-97	170-189	252-266	318-331
F	88-146	255-282	374-394	466-483
G	191-317	552-611	809-852	1,009-1,046
H	250-414	719-796	1,050-1,105	1,694-1,753
I	327-540	938-1038	1,367-1,438	2,879-2,978
J	555-918	1,592-1,762	2,322-2,443	--
K	677-1119	1,941-2,146	2,826-2,973	--
L	826-1365	2,366-2,617	3,444-3,622	--
M	1,008-1,665	2,886-3,191	4,198-4,414	--



zone distances from the station as in Figure 1, it is seen that each selected correction value defines a close approximation to a linear change in elevation with distance. Expressed as slope angles on the basis of the tangent values (elevation/distance), the values are

0.01 mgal	2°52'
0.05 mgal	6°37'
0.10 mgal	9°18'
0.15 mgal	14°7'

These values in turn if used to plot mgals as a function of angle define a linear relation which as shown in Figure 3 can be written

$$Y = .0126 X^{\circ} - 0.025 \text{ mgal} \quad (4)$$

Thus for a 10° angle between the site and adjacent topography out to a distance corresponding to the center of Zone H (5,018-8,578 ft.) along a given azimuth the correction is essentially 0.1 mgal for each zone with a window width of 30°. To get the complete terrain correction out to this distance observations would have to be made along 12 azimuths at 30° intervals about the observation site and their sum used for the total correction. Similarly in Figure 2 the elevation differences for corrections of 0.01, 0.05, 0.10, and 0.15 mgal for each of the Hammer Zones H through M (5,018-71,996 ft.) are plotted

Fig. 1. Elevation Differences for  
Terrain Corrections of 0.01 to 0.15 mgals  
per Zone for Hammer Zones D through H on a Single Azimuth

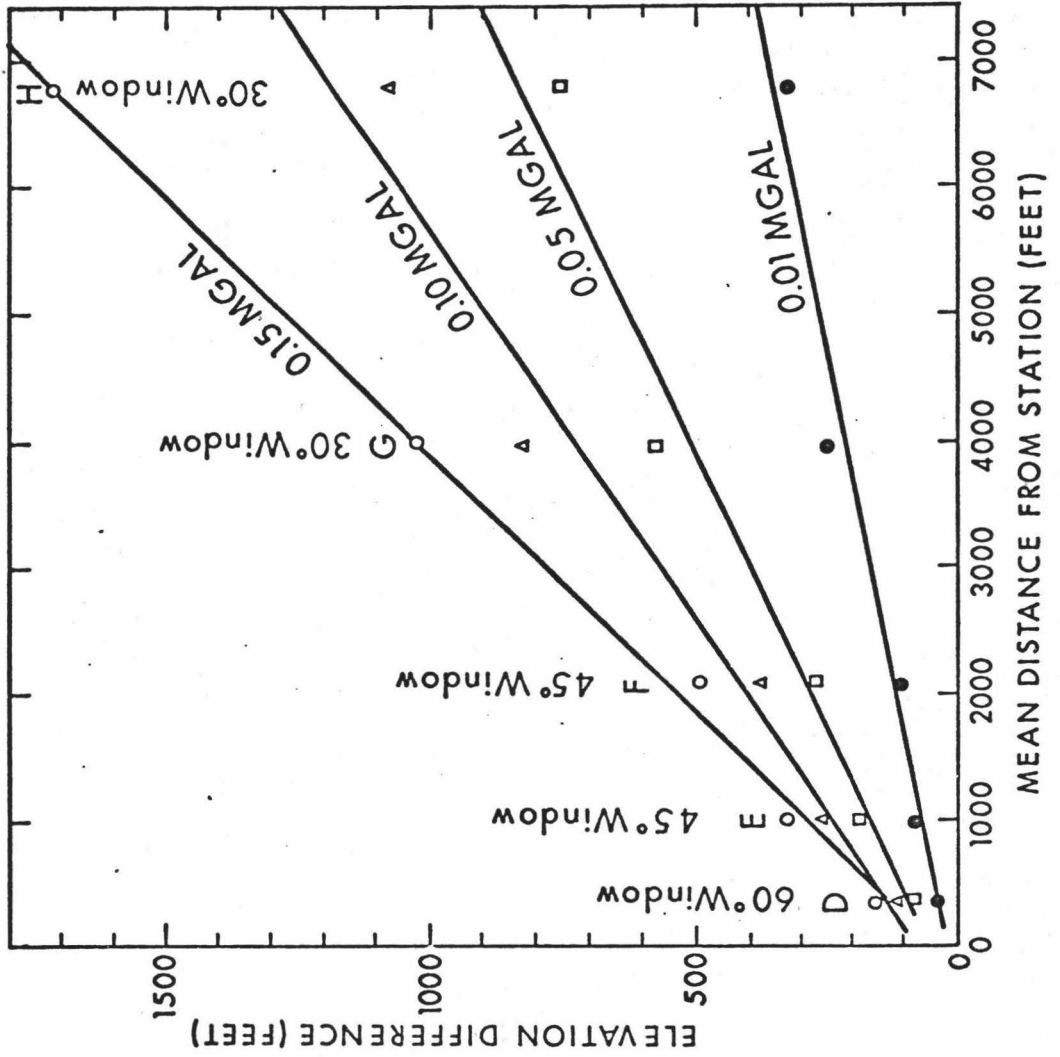
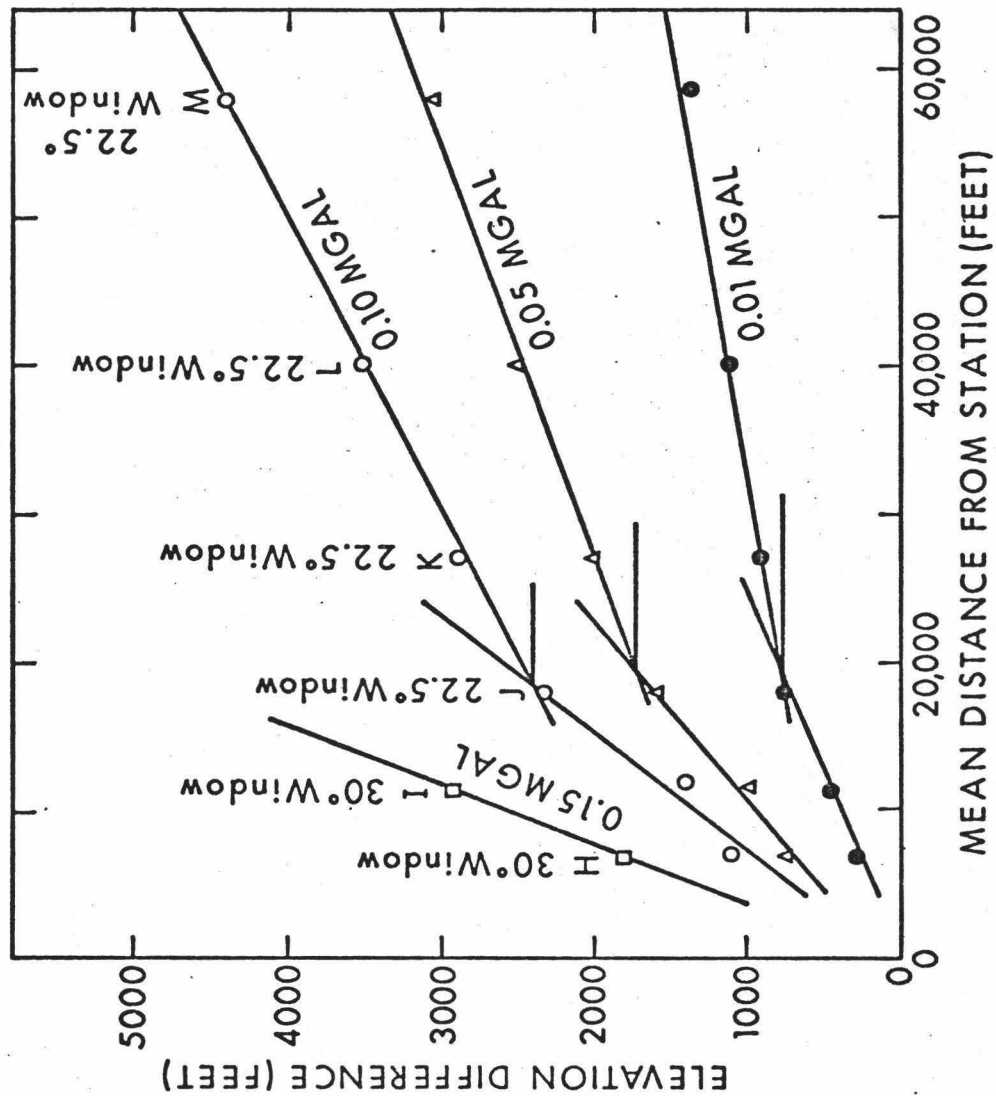


Fig. 2. Elevation Differences for Terrain  
Corrections of 0.01 to 0.15 mgals per Zone for  
Hammer Zones H through M on a Single Azimuth



to define angle relationships at distances greater than that of Zone H from the observation site.

In this plot it will be noted that the data describe two linear relationships intersecting on Zone J at a distance of  $\approx 18,240$  ft. (3.45 miles) from the observation site. As seen, the change in slope is marked, and in each case the change is approximately one half that from Zone H to Zone J. It is also to be noted that the slope from Zone H to J is not the same as that from Zone D to H.

If the equivalent angular values defined are used as before as a base for plotting the terrain correction, it is seen from Figure 3 that two linear relations are defined. The one from Zone H to Zone J is

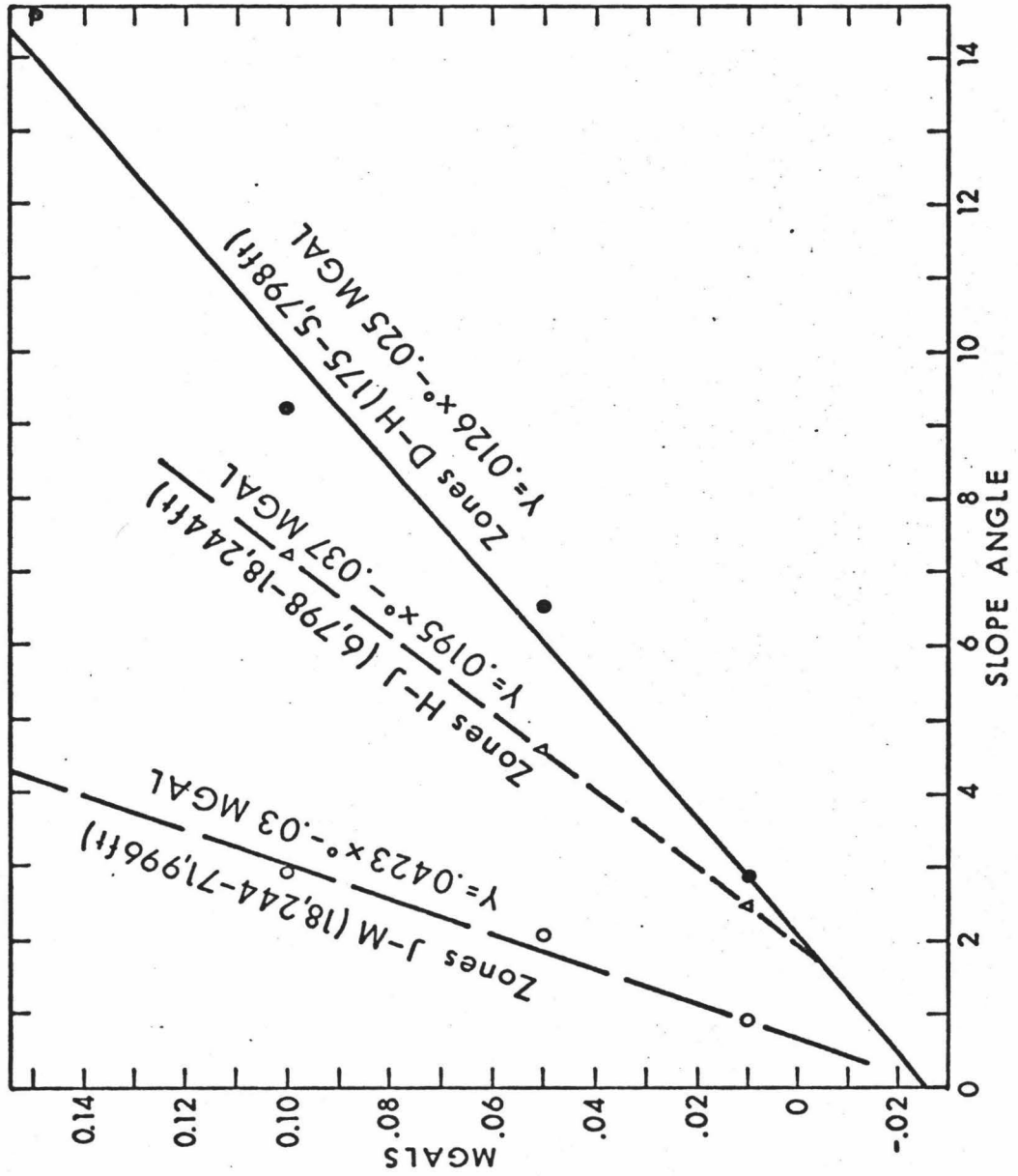
$$Y = .0195 X^\circ - 0.037 \text{ mgal} \quad (5)$$

that from Zone J through Zone M is

$$Y = .0423 X^\circ - 0.03 \text{ mgal} \quad (6)$$

A  $10^\circ$  angle for middle distance topography (Zone H to J) would give a correction of 0.152 mgal per zone on a single azimuth, and as Zones H and I have 12 compartments and Zone J has 16 compartments, the integration of azimuthal values would have to reflect this. Beyond Zone J out through Zone M a  $10^\circ$  angle on more distant topography would result in a terrain correction of  $\approx 0.393$  mgal per zone on a single

Fig. 3. Terrain Correction per Zone  
on a Single Azimuth as a Function  
of Topographic Slope Angle





azimuth, and as these more distant zones have 16 compartments, data would also be required on 16 azimuths.

The total labor in making the terrain correction on the basis of the above would thus involve four groups of determinations representing zones B-D, D-H, H-J, and J-M. In Zone B-D, 6 azimuths would have to be scanned to make sure the topography did not depart by more than 80 ft. from the station elevation to avoid having more than 0.05 mgal terrain effect on any azimuth. In Zone D-H up to 12 azimuths would have to be scanned, and so on.

Although the above would cut down the time required for making a terrain correction since there is no requirement to ascertain the mean elevation of the 134 compartments from Zone A through Zone M around each station, but only the range and elevation of significant changes in elevation from that of the observation site, the procedure as outlined is somewhat cumbersome and does not permit any carry over of information from one station to another in the same area. The method, however, is useful in the field where direct angle measurements on surrounding terrain rising above or falling below the level of the observation site can be made.

A variant of the above is to use the mean angular value for an entire zone surrounding a station, but as brought out by Machesky (1964) this approach results in

values that are consistently too small when results are evaluated against corrections determined using the more rigorous Hayford-Bowie and Hammer zonal methods.

The writer in his approach to the problem has undertaken to devise a method embodying the best features incorporated in the scheme devised by the U. S. Geological Survey (use of a grid system of values in which the values would have application for all stations in an area) and at the same time take advantage of the time saving features of the angular method of study.

#### 5.0 Data Used and Method of Study

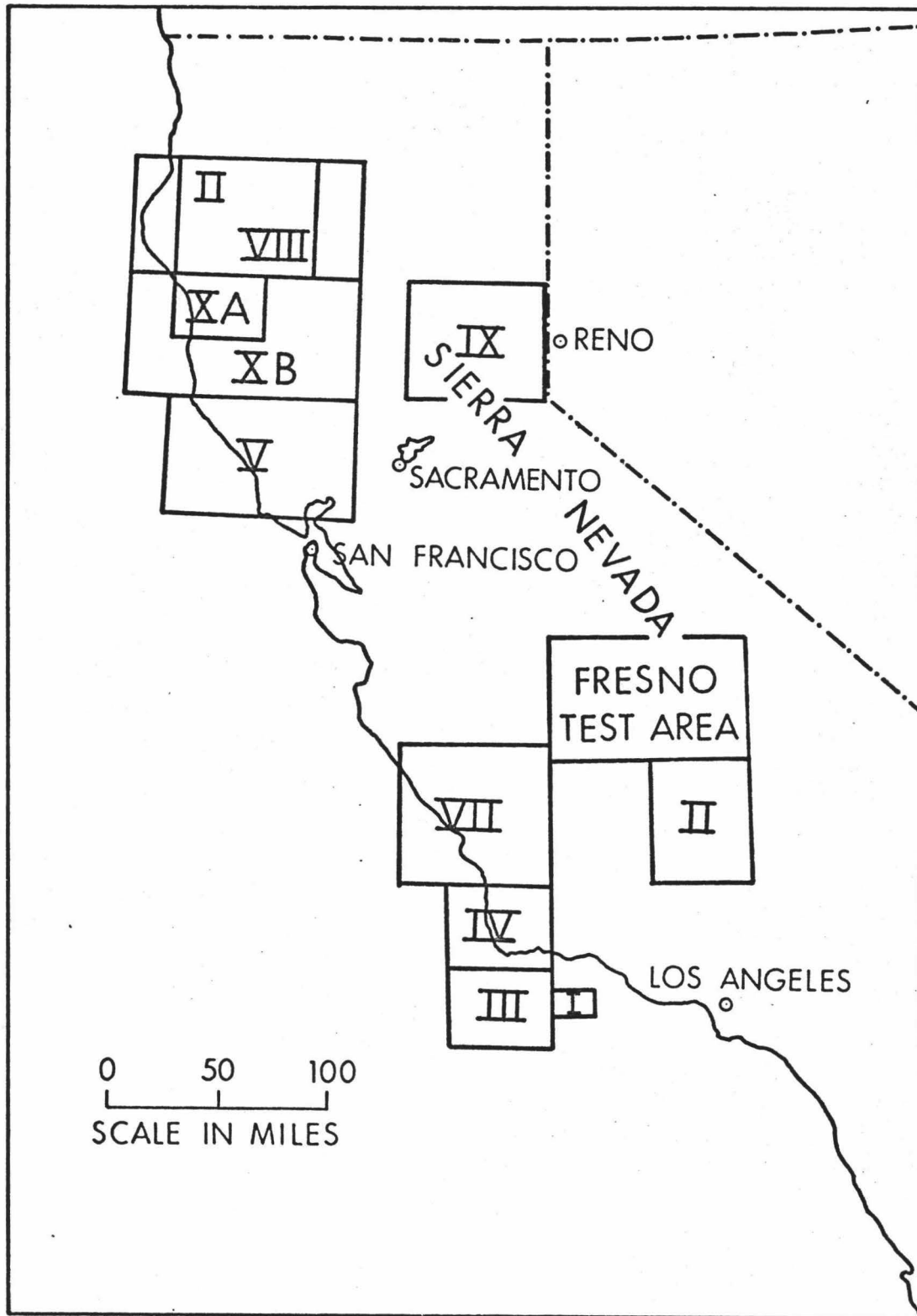
In making this present study a large body of data representing 1,276 stations taken by the U. S. Geological Survey in California (Oliver, 1969) was used. These had all been reduced to yield complete Bouguer anomalies with terrain corrections out to 167 km. This body of data was used, first, to further test the dependence of the terrain correction on topographic slope; then to establish a set of general relationships involving both the topographic slope and elevation difference of the station and regional surrounding terrain, and finally to evaluate the degree of reliability that could be placed in the resulting corrections by testing them against known values in a mountainous area, the Sierra Nevada Mountain region of

California. The location areas for the data used are shown in Figure 4.

#### 5.1 Test of Dependence of the Terrain Correction on Topographic Slope Angle

To test the dependence of the terrain correction on the topographic slope angle, especially that near the station out to 2 km distance, topographic slope values to the nearest topography rising above or falling below the elevation of the station in the immediate environs of the station (distances of 1 and 2 km corresponding approximately to the mid-point radii of terrain correction Zones E and F (see Table 2), respectively, were used. The choice of these two distances as being diagnostic is evident from Table 3 in that the compartment zonal correction values for elevation differences in Zones E and F have maximum values for what might be regarded as probable maximum elevation differences (3,000 to 8,000 ft.) that might be encountered at these distances. As seen from Table 3, at greater distances the same elevation differences give significantly smaller corrections. For example, in Zone G with a mean radius of 2.8 km the correction for an elevation difference of 3,500 ft. is less than half of that in Zone F and only about one sixth that in Zone E.

Fig. 4. Locations of Areas in  
Which the U. S. Geological Survey had  
Established Terrain Corrections for the 1276  
Gravity Stations Used



The argument for evaluating the relation of the total terrain correction in terms of the slope along one azimuth (the direction of maximum slope) can be justified on the basis of the overall linear strike of most major topographic features and changes in regional elevation. This is particularly true of the topography in the test evaluation area which parallels the coast line of California, and although there are parallel ranges with reversals in dip, over the short distances involved (1 to 2 km) the direction of maximum slope is that of an east to west or west to east dipping plane surface. In general, therefore, the slope of the topography on other azimuths out to  $90^\circ$  on either side of the direction of maximum slope bear a more or less systematic relation between that in the direction of maximum slope and that along strike where the slope is zero. Whether the slope is positive or negative is not important since the sign of the gravity effect is always the same.

A third argument, and one which pertains in particular to not using distances less than 1 km in measuring the topographic slope, is that except on the edge of a canyon or cliff, both unlikely places for making gravity observations, the slope out to the mid-point of Zone D (300 meters) is likely to continue essentially uninterrupted out to 1 km and more.

The only type of area in which using the above procedure will result in probable large errors and scatter in values showing a poor correlation between slope angle and terrain correction is in a mountain crestal zone where there has been extensive glaciation with resulting cirques, hanging valleys, comb and similar non-systematic high relief topography that does not conform to any general overall strike pattern.

It was on the basis of the above rationale plus the fact that a terrain correction scheme based on an inclined plane (Sandberg, 1958) had proved to give results comparable to a rigorous solution over a radius of 20 km in many areas, that the writer carried out phase one of the present study. The objectives were to determine the degree that a systematic empirical relationship could be established using the maximum slope angle and the conditions under which such an empirical relationship might break down.

#### 5.1.1 Method of Measurement Used and Results

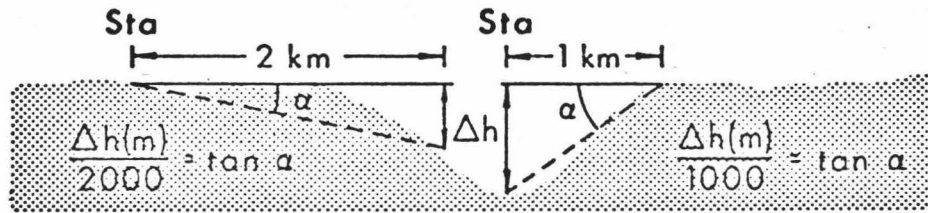
The determination of whether to use 1 or 2 km distance in determining the topographic slope along the direction of maximum slope was based upon the distance giving a continuous maximum slope angle. If the slope was less beyond 1 km, the measurement was only made to 1 km distance. If it continued or increased out to 2 km, that distance was used.

In measuring the slope where the observation site was not located on a continuous topographic slope, as on a mesa top back from its edge or the side of an incised valley, or on the crest of a hill, or in the center of a broad valley, the slope used was that from the observation site to the nearest point defining the maximum change in elevation and at the same time giving the closest approximation of equal topographic mass excess and deficiency above and below the slope line. See Figure 5 which illustrates the procedure used in determining the slope under these conditions. As an aid in determining the maximum slope values, a transparent template with inscribed circles having radii of 1 and 2 kms on a scale of 1:250,000 was used that could be centered on the station and the azimuth and value of maximum slope quickly estimated by visual inspection and counting of contour lines.

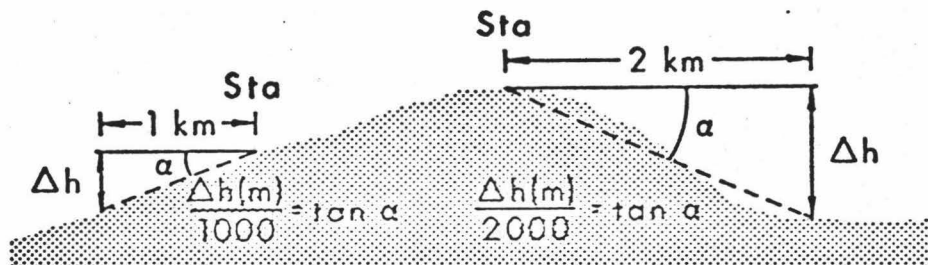
Although topographic maps on a scale of 1:25,000 were available for much of the area covered by the gravity data being used, a decision was made to use the AMS 1:250,000 scale topographic maps with 200-foot contours for this preliminary study. The reason being that for much of the United States these are the best topographic maps available, and it was felt that to demonstrate significance in the results the test should be made under conditions and handicaps that would prevail in areas where there were no better maps.



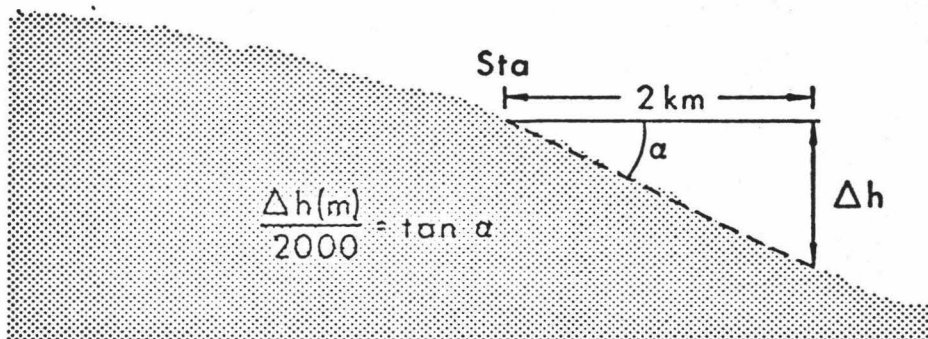
Fig. 5. Method Used in  
Establishing Maximum Topographic  
Slope for Sites not on a Slope Face



Valley Station  
and  
Side Wall Valley Station



Ridge Crest Station  
and  
Ridge Flank Station



Slope Station

In Table 6 the data are tabulated in terms of groups of stations having essentially the same slope values as defined by tangent values for the topographic slope angle in the direction of maximum slope above or below the gravity observation sites. These are subdivided on an arbitrary basis into approximately 0.015 increments corresponding to angular changes ranging from  $2^{\circ}40'$  to  $38^{\circ}25'$ . For each group increment value the average terrain correction, number of values averaged and spread in values found for a given slope increment are also given. In Figure 6 the average terrain correction values for each group are plotted as a function of the slope tangent values.

As seen from Figure 6, a smooth curve can be fitted to the data points that would define the terrain correction to better than 2 mgal for 82 percent of the values plotted representing 98 percent of the total volume of data represented. Although the scatter in values is significantly larger (4 to 5 mgals) once the slope tangent value exceeds 0.5 corresponding to an angle of  $26.5^{\circ}$ , this uncertainty applies to only about half (4 out of 7) of the data points for slope angles greater than  $26.5^{\circ}$ . For slope angles less than  $26.5^{\circ}$  only 2 out of the 21 data points plotted show a similar degree of departure from the mean, and in both cases the actual terrain correction was larger than that depicted by the best fit curve. That these

TABLE 6

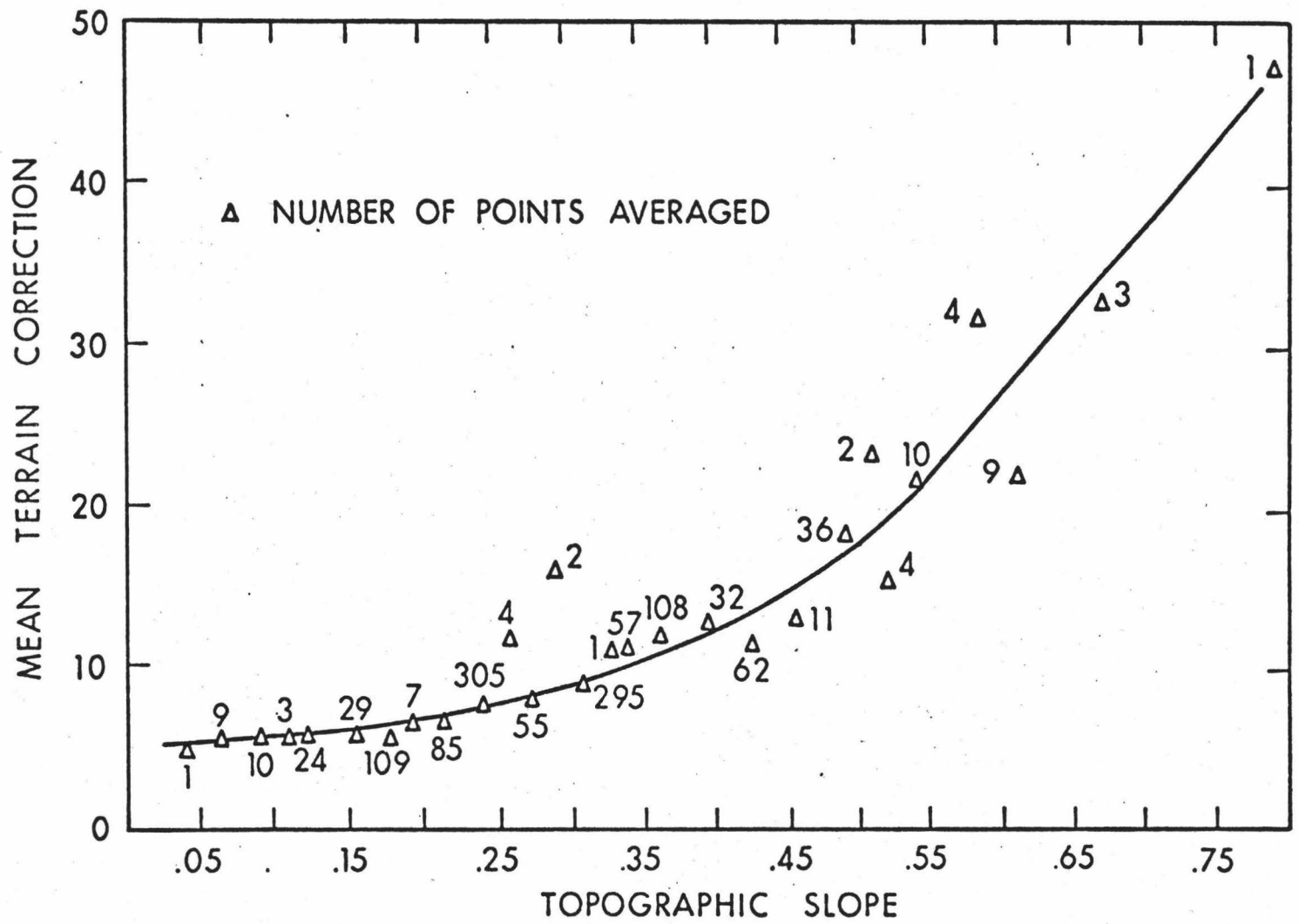
TERRAIN CORRECTION VALUES FOR GROUPS  
OF STATIONS HAVING ESSENTIALLY THE SAME  
MAXIMUM TOPOGRAPHIC SLOPE OUT TO 1 OR 2 KM  
FROM THE OBSERVATION SITE

Degree of Slope	Number of Stations	T. C. Range	Mean T. C. n
.046	1	5	5
.061	9	5-8	6
.092	10	5-10	6
.107	3	5-6	6
.122	24	5-10	6
.152	29	5-9	6
.183	109	5-21	6
.198	3	5-9	7
.214	85	5-12	7
.244	303	5-18	8
.260	4	9-16	12
.274	55	5-24	8
.289	2	13, 18	16
.305	295	5-25	9
.320	1	11	11
.335	57	5-23	11
.366	108	5-38	12
.396	32	5-21	13
.427	62	5-32	12

TABLE 6. (Continued)  
 TERRAIN CORRECTION VALUES FOR GROUPS  
 OF STATIONS HAVING ESSENTIALLY THE SAME  
 MAXIMUM TOPOGRAPHIC SLOPE OUT TO 1 OR 2 KM  
 FROM THE OBSERVATION SITE

Degree of Slope	Number of Stations	T. C. Range	Mean $\frac{T. C.}{n}$
.457	11	6- 24	14
.488	36	7- 29	18
.503	2	23, 25	24
.518	4	11- 22	15
.549	10	5- 36	22
.579	4	22- 43	32
.610	9	13- 42	23
.649	0	--	--
.671	3	29- 39	33
.793	1	46	46

Fig. 6. Plot of Average Terrain  
Correction Versus Average Tangent Value  
of Maximum Topographic Slope Angle Out to 1 and 2 kms



two "poor" values represent extraordinary situations is indicated by the fact that only 6 stations are involved, with the greatest departure (8 mgals) being associated with only 2 stations.

The results of the preliminary study therefore indicated that on an overall basis the terrain correction could be established quickly on a simple empirical basis with an average reliability of better than 2.0 mgal at least 90 percent of the time as long as the maximum topographic slope angle did not exceed  $26^{\circ}$ . For topographic slope angles greater than  $26^{\circ}$ , it could be expected that half of the time the average terrain correction deduced would be 5 to 6 mgals in error.

#### 5.2 Effect of Considering Deviation from Regional Elevation

As the results obtained in the initial phase of study did not reflect the actual spread in values and it appeared they could be improved upon if the mean elevation of the topography around a station were taken into consideration, the second phase of the study, based on the same body of gravity data was initiated.

As before, and for the same reasons given earlier, maps on a scale of 1:250,000 with a contour interval of 200 feet were used to establish a grid system of mean elevation values. The unit grid size used as building

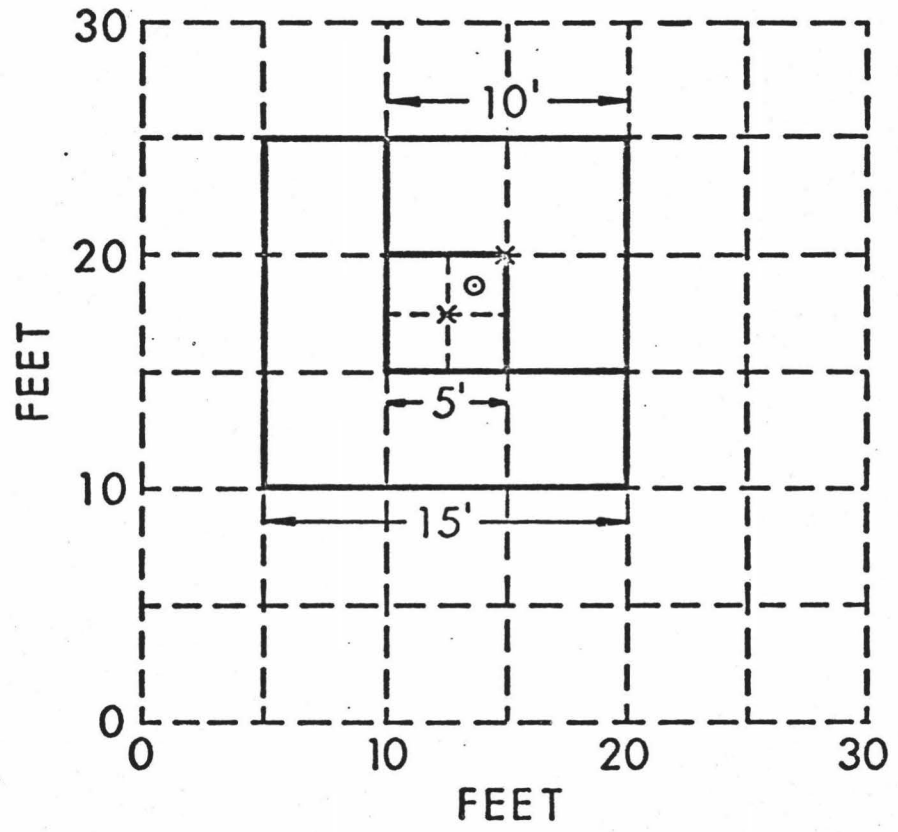


blocks were 5 x 5 minute squares, and the average elevation for each such square was computed on the basis of a 10 point representation of elevation values.

These were the 4 corner points, the 4 mid points on each side and the center point taken with a value of two. This system of determining average elevation values has been found to have a high degree of reliability for areas of this size and has been extensively used for defining both average areal elevation and geophysical anomaly values. As shown in Figure 7 with these values established on a 5' x 5' basis no station will lie further than 2.5' from the center of such a square or that of a 10' x 10', 15' x 15' or larger square having dimensions of 30' x 30' or 1° x 1°. As is evident, the mean elevations of such larger areal units can be quickly determined from the 5' x 5' unit values by simple averaging of the appropriate 5' x 5' values surrounding the station square.

For the center 5' x 5' block, which is approximately 9.27 km on a side, a template was constructed that subdivided the area into twenty-five 1' x 1' squares. Visual estimates of elevation for each of the 24 squares surrounding the center (station) square could then be made quickly, and by using the "corner values for the center station square, a more precise 5' x 5' mean value obtained for this square. This was felt to be desirable because of the particular

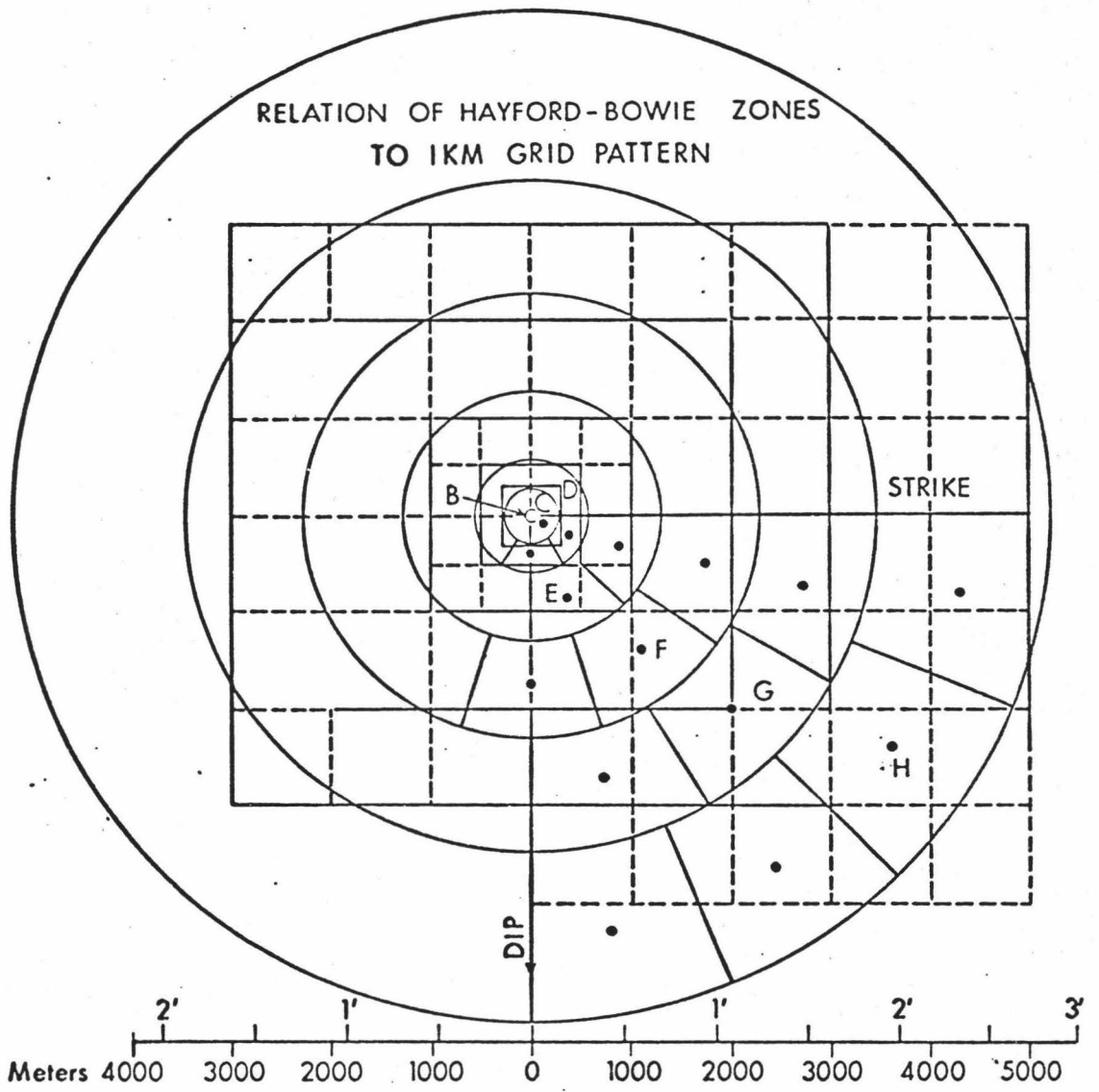
Fig. 7. Station Square Location  
Relative to a Surrounding Regional Net of  
5' x 5' Size Areas where Elevation is Known



significance of this square in making the terrain correction. This is brought out in Figure 8 in which the Hayford-Bowie zones through Zone H, corresponding approximately to the center distance of a 5' x 5' square, are superimposed upon a 1 x 1 km grid system. It will also be noted that the center 1' x 1' square corresponding closely to a 2 x 2 km square are taken in the mean radius value for Zone F.

With these values established, the difference in station elevation relative to the surrounding average elevation values for squares of 5' x 5', 10' x 10', 15' x 15', 30' x 30', and 1° x 1° size were examined in terms of possible correlations with the terrain corrections for the stations. No correlation was evident in the data for the 30' x 30' and 1° x 1° size squares, but a correlation was evident in the data for the 10' x 10' and 15' x 15' size squares, and in areas of uniform slope also with the data for the 5' x 5' size squares. These results agree with those of Groten and Reinhart (1968) who concluded from their study that in general the topographic correction is due to the topography lying within a radius of 10 to 30 kms of the station. This of course is also brought out in Table 3 and is implied in the Hammer terrain correction tables which stop at a radius of 22 kms with an elevation difference of 4300 feet relative to that of the observation site.

Fig. 8. Relation of Hayford-Bowie Zones  
to a Kilometer Square Grid Pattern and  
Center 5' x 5' Square



The next step was to try to systematize these observations in terms of the maximum slope angles out to 1 and 2 km and the difference in elevation of the station relative to that of the surrounding terrain out to  $\approx 7.5'$  (14 km) around the station.

In effect, the approach is a variant on that tried by Machesky (1964) in which he considered the station as lying at the apex of a cone rising above the surrounding terrain where the station elevation was greater than that of the surrounding terrain, or as being at the apex of an inverted cone incised below the regional elevation when the station elevation was less than the regional value. Machesky, however, in using the slope relationships defined by average elevations for individual complete zones surrounding a station found he invariably obtained values that were significantly smaller than true values rigorously derived. What was needed was therefore some method to compensate for this defect in his method.

As a basis for a first trial and test, the data for 1243 of the stations available was organized along the following lines:

- (1) The terrain corrections.
- (2) The relative difference in elevation of the station and that of the surrounding 15' x 15' square, and

differentiating positive and negative halves where a slope was involved.

(3) The maximum slope angle found in the first 1 to 2 km distance from the station expressed as tangent values. These data were then segregated on the basis of these stations having essentially the same maximum slope tangent values out to 1 and 2 km in increments of  $\approx 0.03$ . The observed terrain correction values for each group were then plotted as a function of the difference ( $\Delta h$ ) in station elevation value and the regional 15' x 15' elevation value.

As seen from Figure 9, a series of more or less regular conformable curves was obtained for the data plotted in this fashion indicating that the approach had considerable merit.

The next step was to derive a model that would give an equivalent set of curves that would have general application and not reflect the idiosyncracies of abnormal topographic relations evident in the empirical relations defined in Figure 8.

#### 6.0 Model Developed for Determining the Terrain Correction

As shown in Figure 10, the zonal method of correcting for terrain will clearly give a representation of a conical hill or depression. It was therefore logical to base the model on the gravitational attraction on a cone



Fig. 9. Plots of Terrain Correction Observed  
Versus Elevation Difference of Station and  
Surrounding Terrain in a 15' x 15' Square for  
Stations having the Same Maximum Topographic  
Slope within 1 to 2 kms of Station

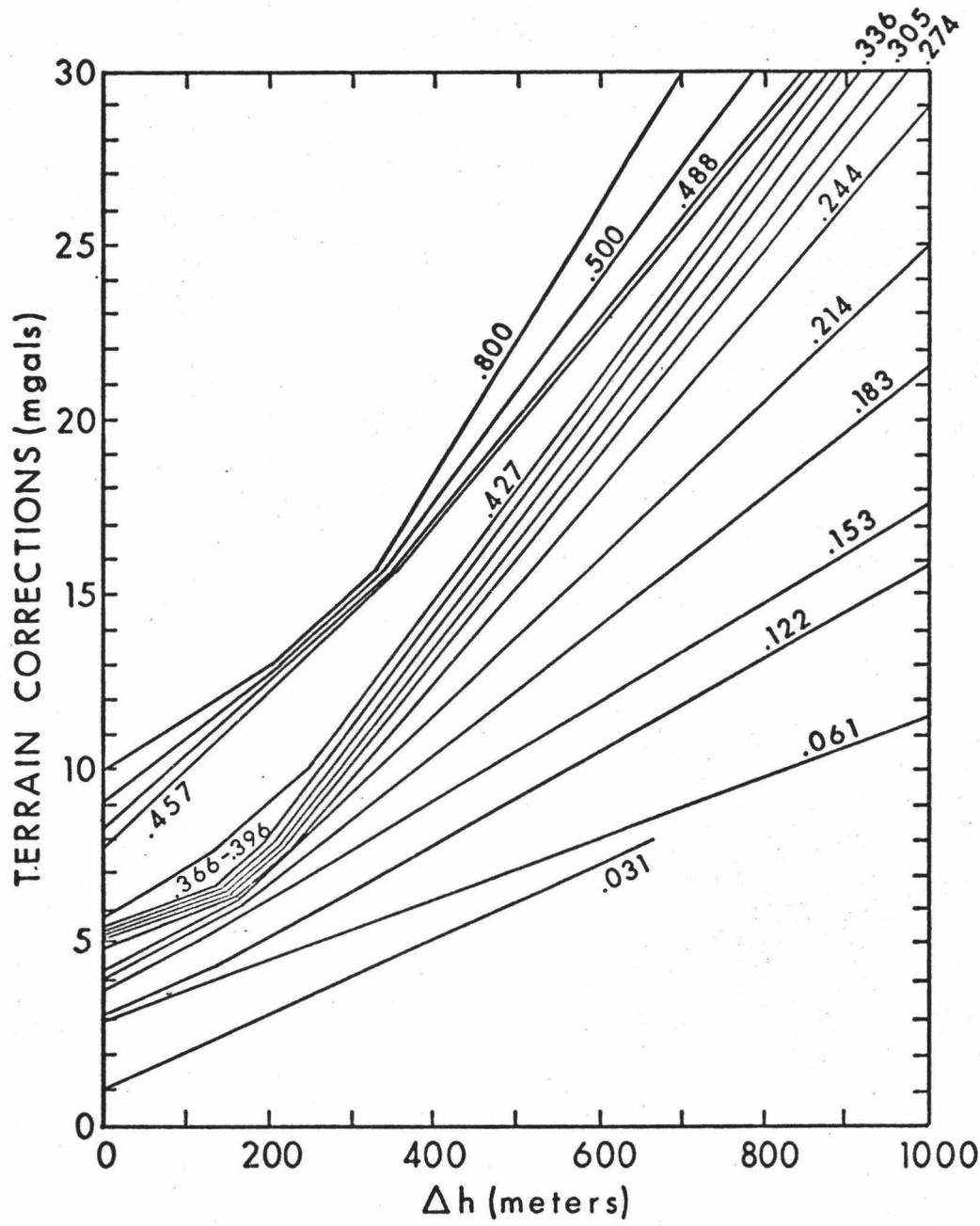
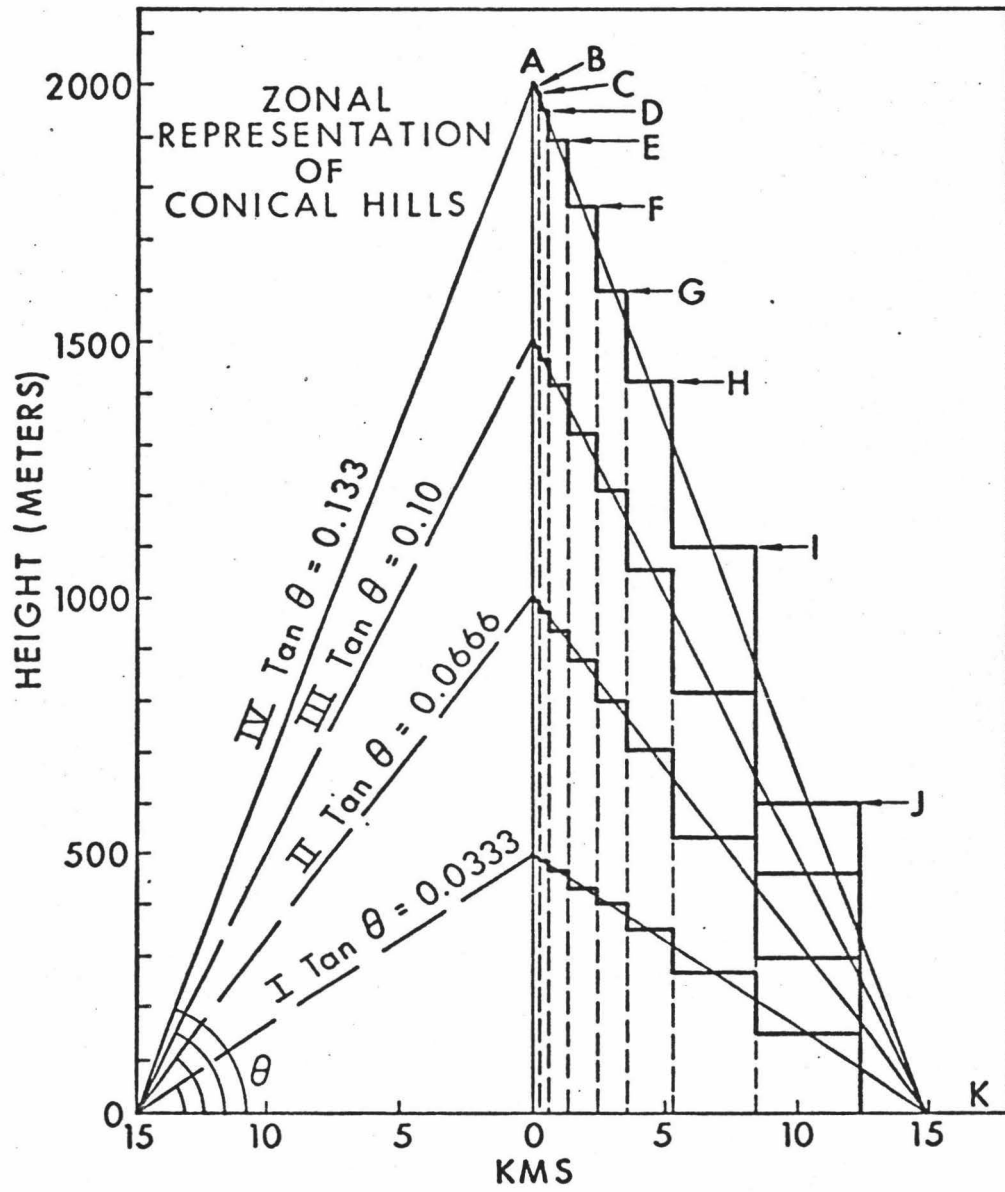


Fig. 10. Relation of Hayford-Bowie Zones to a  
Conical Hill with a Base Width of 30 kms  
and a Height of 2000 Meters



whose height would be related to the elevation difference ( $\Delta h$ ) of the station and the surrounding terrain, and with a slope angle reflecting the dominant gravitational effect of the topographic change within 1 to 2 km (Hayford-Bowie Zones A to F) of the station. On a ridge or in a valley, the two dimensional nature of the topography is automatically compensated for by the decrease in the value of  $\Delta h$  relative to that which would be had for a conical hill or depression with the same total relief. On a topographic slope where the mean value for the regional elevation could equal that of the station, it is only necessary to consider the topographic correction as having two parts, that for the topography lying above the level of the station and that for the topography lying below the station and using the sum of these two partial components contributing to the terrain correction.

The scheme developed, therefore, was based on the difference in the gravitational attraction of a plate and that of a cone having the same elevation ( $\Delta h$ ), and with a slope angle ( $\Delta h$ ) equal to that for the maximum slope out to 1 or 2 km from the station depending on which gave the greater value. The basic expression for the terrain correction (TC) neglecting the small curvature correction, therefore, can be written as

$$TC = \Delta g_s - \Delta g_c \quad (7)$$

where  $\Delta g_s$  (the plate attraction) =  $0.1118 \Delta h$  (meters) and

$$\Delta g_c \text{ (the cone attraction) } = 0.1118 (\Delta h - \sin\alpha\Delta h)$$

$$\text{and TC} = 0.118 \sin\alpha\Delta h \quad (8)$$

The only variant of equation (8) would be where the topography both above and below the station has to be considered. Under these conditions, each portion can then be considered as a half cone solution as the mean regional elevations used in defining the respective values of  $\Delta h$  are of equal size.

Although most isolated topographic features, other than mountain ranges as a whole have a base width less than 20 km, a test was made of the terrain correction that would be obtained using equation (8) for a conical hill with a base width of 30 km rising 2000 meters above a level plane as compared to that obtained using the Hayford-Bowie terrain tables of Swich (1942) carrying the corrections out to a radius of 166.7 km. This is the same model shown in Figure 10.

For  $\Delta h = 2$  km and  $b/2 = 15$  km, the slope angle tangent is  $2/15 = 0.1333$ , and  $\sin \alpha = 0.1321$

Substitution in  $\Delta g_c = 0.1118 (\sin \alpha \Delta h)$

$$\Delta g_c = 0.1118 (0.1321 \times 2000) = 28.6 \text{ mgals}$$

Solving for the terrain correction using Table 3, the data are as follows:

Zone	Mid Dist km	% of K Value	Elv Diff m	Corr* $\times 10^{-5}$ gal	Corr $\Sigma$ $\times 10^{-5}$ gal
A	0	0	0	0	0
B	.034	0.22	4	$7 \times 4 = 28$	28
C	.149	0.95	19	$5 \times 4 = 20$	48
D	.410	2.62	52	$7 \times 6 = 42$	90
E	.935	6.0	120	$9 \times 8 = 72$	162
F	1.79	11.5	230	$9 \times 10 = 90$	252
G	2.91	18.6	373	$11 \times 12 = 132$	384
H	4.38	28.2	562	$10 \times 10 = 160$	544
I	6.84	43.9	855	$15 \times 20 = 300$	844
J	10.42	67.0	1340	$28 \times 16 = 448$	1292
K	15.60	100.0	2000	$33 \times 20 = 660$	1852
L	23.8		2000	$19 \times 24 = 456$	2308
M	43.8		2000	$35 \times 14 = 490$	2798
N	79.9		2000	$27 \times 16 = 432$	3230
O	132.8		2000	$0 \times 28 = 0$	$3230 = 32.30$ mgals

\*Corr = compartment value x number of compartments  
in zone.

The comparative values are thus 28.6 mgals vs 32.3 mgals--a difference of 3.7 mgals. However, it is to be noted that if the terrain correction had been stopped at zone K where the full value of topographic relief was reached, the cone solution would have been 10.1 mgals larger than the zonal method value rather than 3.7 mgals smaller as was found on carrying the zone corrections out through zone N with a mean radius of 79.9 kms from the station.

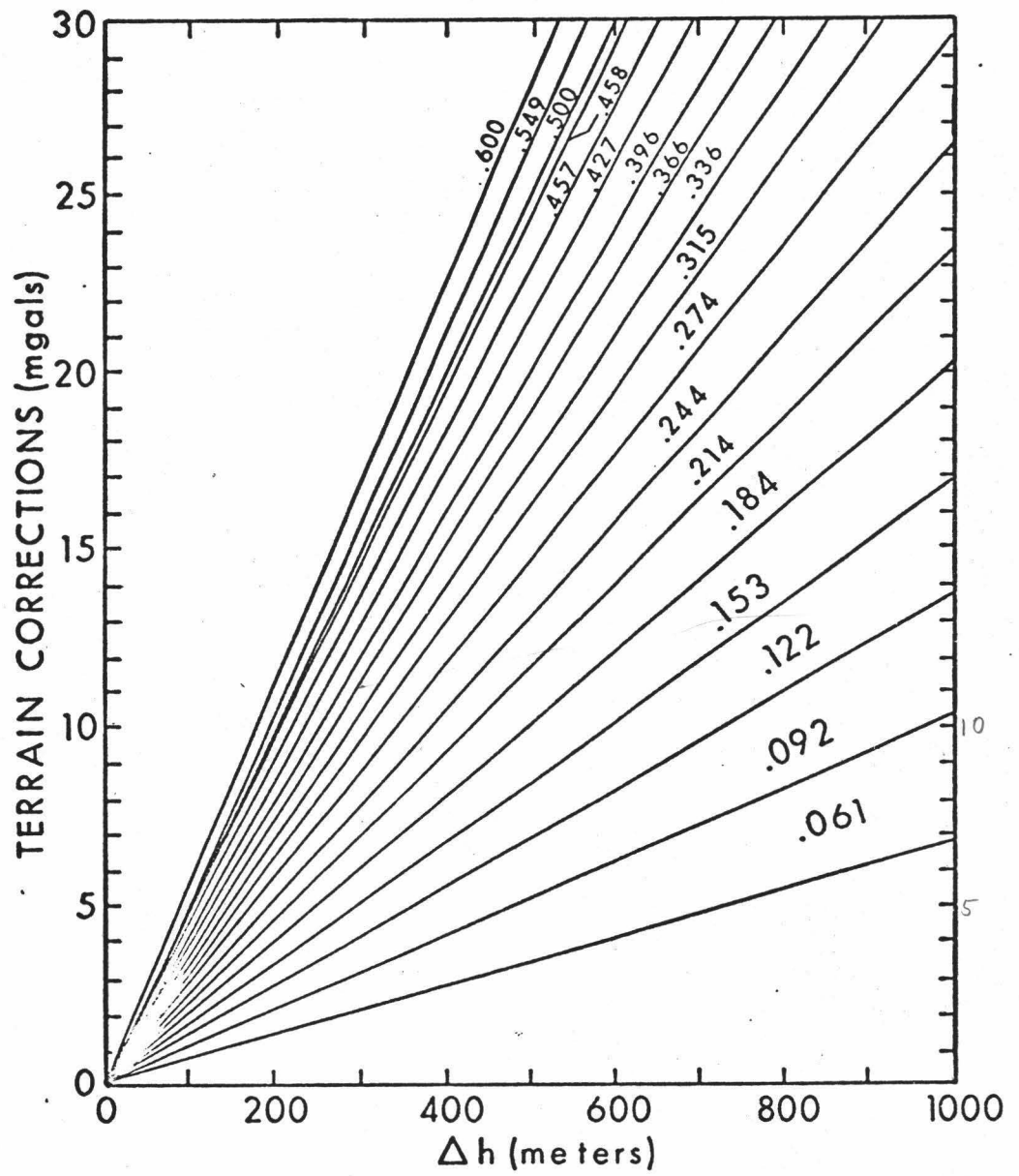
As the example used represents what might be regarded as an extreme case, both in terms of base width and height of a topographic feature normally encountered, the cone solution appeared to be an acceptable alternate method for determining the terrain correction within the 5 mgal level of reliability being sought.

#### 6.1 The Theoretical Curves for Determining the Terrain Correction

On the basis of the above, a series of incremental angle tangent values ranging from 0.061 to 0.60 were used to calculate theoretical terrain correction values for cones having  $\Delta h$  values ranging from 0 to 1000 meters. These curves are shown in Figure 11. The quantities to be used with these curves for determining the terrain correction are:



Fig. 11. Theoretical Terrain Correction Curves  
for Different Slope Tangent Values  
and Values of  $\Delta h$



(1) the tangent of the maximum slope angle observed out to either 1 or 2 km from the station depending on which distance gives the greater value,

(2) the elevation difference ( $\Delta h$ ) between the station and that for the surrounding 15' x 15' (28 x 28 km) square where there is no marked regional slope, or if there is a slope, the value for the (7.5' x 15') half square areas lying above and below the station which are treated separately.

Using these curves and the values for  $\tan \alpha$  and  $\Delta h$  one can interpolate between the plotted curve of Figure 11 to obtain the terrain correction, and as indicated where the station is on a regional slope, half of the terrain correction indicated for each is taken, and the two then added to obtain the total correction.

#### 7.0 Test of Method

To test the reliability of the method outlined in section 6.0, that portion of the U. S. Geological Survey data (256 stations) lying in the Sierra Nevada Mountain area adjacent to Fresno, California was used. This test area, which is roughly 50 x 120 km in size, includes Mount Whitney rising to 14,494 ft. above sea level as well as a variety of high relief erosional features (canyons, cirques, comb ridges, isolated peaks) and also low relief foot hill topography. The test standard was the rigorously

determined terrain corrections for the 256 U. S. Geological Survey gravity stations in the area. These provided both broad regional and elevation coverage including the summit of Mount Whitney and the bottom of Kern River Canyon. The range in terrain correction values is from 1 to 74 mgal, and the correction for 92 sites (36 percent of the total) exceeds 15 mgals.

#### 7.1 Results of Test in Sierra Nevada Mountain Area

In Table 7 the terrain correction as determined on the basis of Equation (8) and as derived by the U. S. Geological Survey for each of 256 stations in the test area are tabulated along with the difference in the two values.

The overall breakdown on comparisons is as shown in Table 8.

As seen from Table 8, 54 percent of the predictions are better than 2 mgals; 79 percent better than 5 mgals and 90 percent better than 8 mgals.

If the data are examined in terms of bias in sign of the differences, and percentage error as a function of the magnitude of the correction, the data are as shown in Table 9.

As brought out in Table 9 there is no consistent bias in sign between predicted and observed values. For

TABLE 7

COMPARISON OF COMPUTED TERRAIN CORRECTIONS  
 BASED ON A CONE WITH ACTUAL VALUES  
 IN SIERRA NEVADA MOUNTAINS AREA

Station Number	USGS Terrain Correction	T.C. = $111.9\Delta h$ $\sin \alpha$	$\Delta$ T.C. USGS Empirical T.C.
1	23	27	4
2	22	23	1
3	14	12	2
4	5	5	0
5	8	7	1
6	12	10	2
7	12	10	2
8	8	6	2
9	9	11	2
10	11	10	1
11	9	9	0
12	7	6	1
13	10	10	2
14	12	14	2
15	7	2	5
16	7	3	4
17	41	40	1
18	9	7	2
19	21	23	2
20	25	28	3
21	30	29	1
22	32	30	2
23	31	28	3
24	13	12	1
25	43	48	5
26	21	21	0
27	28	29	1
28	44	31	13
29	26	23	3
30	22	21	1
31	21	21	0
32	24	20	4
33	34	27	7
34	22	37	15
35	14	16	2
36	14	16	2
37	21	20	1
38	25	30	5
39	15	14	1

TABLE 7. (Continued)  
 COMPARISON OF COMPUTED TERRAIN CORRECTIONS  
 BASED ON A CONE WITH ACTUAL VALUES  
 IN SIERRA NEVADA MOUNTAINS AREA

Station Number	USGS Terrain Correction	T.C. = $111.9\Delta h$ $\sin \alpha$	$\Delta$ T.C. USGS Empirical T.C.
40	19	17	2
41	11	4	7
42	7	6	1
43	8	2	2
44	12	8	4
45	20	21	1
46	22	33	11
47	17	15	2
48	74	75	1
49	34	34	0
50	45	32	13
51	17	11	6
52	14	9	5
53	12	5	7
54	20	25	5
55	16	21	5
56	22	31	9
57	18	21	9
58	18	15	3
59	6	5	1
60	21	23	2
61	44	44	0
62	9	7	2
63	24	28	4
64	13	9	4
65	12	8	4
66	17	16	1
67	50	46	4
68	21	23	2
69	16	11	5
70	16	15	1
71	39	35	4
72	14	9	5
73	15	17	2
74	15	9	6
75	16	17	1
76	12	9	3
77	6	7	1
78	10	7	3
79	30	37	7

TABLE 7. (Continued)  
 COMPARISON OF COMPUTED TERRAIN CORRECTIONS  
 BASED ON A CONE WITH ACTUAL VALUES  
 IN SIERRA NEVADA MOUNTAINS AREA

Station Number	USGS Terrain Correction	T.C. = $111.9\Delta h$ $\sin \alpha$	$\Delta$ T.C. USGS Empirical T.C.
80	9	4	5
81	12	12	0
82	15	13	2
83	11	11	0
84	14	10	4
85	20	17	3
86	21	24	3
87	19	19	0
88	17	13	4
89	15	13	2
90	15	16	1
91	15	16	1
92	48	38	10
93	27	15	12
94	13	11	2
95	17	23	6
96	18	10	8
97	35	40	5
98	13	21	8
99	14	21	7
100	15	10	5
101	14	10	4
102	11	9	2
103	17	24	7
104	13	18	5
105	18	25	7
106	16	13	3
107	25	25	0
108	15	13	2
109	12	21	9
110	11	19	8
111	14	18	4
112	14	14	0
113	16	28	12
114	32	32	0
115	15	20	5
116	10	14	4
117	13	11	2
118	34	25	9
119	33	37	4

TABLE 7. (Continued)  
 COMPARISON OF COMPUTED TERRAIN CORRECTIONS  
 BASED ON A CONE WITH ACTUAL VALUES  
 IN SIERRA NEVADA MOUNTAINS AREA

Station Number	USGS Terrain Correction	T.C. = $111.9\Delta h$ $\sin \alpha$	$\Delta$ T.C. USGS Empirical T.C.
120	35	45	10
121	28	23	5
122	27	21	6
123	7	10	3
124	5	9	4
125	9	11	2
126	10	12	2
127	20	27	7
128	14	17	3
129	13	14	1
130	12	15	3
131	13	18	5
132	21	31	10
133	20	30	10
134	13	18	5
135	10	16	6
136	13	20	7
137	11	11	0
138	11	10	1
139	10	8	2
140	10	7	3
141	18	11	7
142	37	51	14
143	29	22	7
144	3	4	1
145	4	3	1
146	3	3	0
147	4	4	0
148	4	3	1
149	6	7	1
150	4	3	1
151	5	5	0
152	8	14	6
153	7	9	2
154	12	19	7
155	12	13	1
156	6	8	2
157	12	21	9
158	10	16	6
159	12	18	6



TABLE 7. (Continued)  
 COMPARISON OF COMPUTED TERRAIN CORRECTIONS  
 BASED ON A CONE WITH ACTUAL VALUES  
 IN SIERRA NEVADA MOUNTAINS AREA

Station Number	USGS Terrain Correction	T.C. = $111.9\Delta h$ $\sin \alpha$	$\Delta$ T.C. USGS Empirical T.C.
160	12	16	4
161	12	24	12
162	12	19	7
163	10	15	5
164	11	2	9
165	10	12	2
166	8	9	1
167	10	15	5
168	12	10	2
169	13	14	1
170	30	32	2
171	11	10	1
172	22	29	7
173	18	24	6
174	26	34	8
175	30	22	8
176	44	41	3
177	11	10	1
178	10	6	4
179	3	3	0
180	3	4	1
181	5	4	1
182	3	2	1
183	4	3	1
184	6	5	1
185	4	4	0
186	4	2	2
187	6	4	2
188	5	2	3
189	7	7	0
190	5	4	1
191	12	10	2
192	7	9	2
193	7	7	0
194	15	14	1
195	8	11	3
196	11	16	5
197	9	12	3
198	11	15	4
199	21	32	11

TABLE 7. (Continued)  
 COMPARISON OF COMPUTED TERRAIN CORRECTIONS  
 BASED ON A CONE WITH ACTUAL VALUES  
 IN SIERRA NEVADA MOUNTAINS AREA

Station Number	USGS Terrain Correction	T.C. = $111.9\Delta h$ $\sin \alpha$	$\Delta$ T.C. USGS Empirical T.C.
200	25	25	0
201	10	9	1
202	20	15	5
203	9	10	1
204	3	1	2
205	11	12	1
206	3	3	0
207	5	7	2
208	6	5	1
209	7	11	4
210	14	23	9
211	9	10	1
212	9	12	3
213	10	10	0
214	28	44	16
215	22	40	18
216	14	19	5
217	17	23	6
218	15	20	5
219	19	28	9
220	3	3	0
221	4	4	0
222	3	3	0
223	5	4	1
224	5	4	1
225	11	9	2
226	6	4	2
227	4	2	2
228	4	6	2
229	23	29	6
230	9	9	0
231	22	23	1
232	6	5	1
233	16	31	15
234	19	28	9
235	2	1	1
236	2	1	1
237	4	2	2
238	3	1	2
239	4	2	2

TABLE 7. (Continued)  
 COMPARISON OF COMPUTED TERRAIN CORRECTIONS  
 BASED ON A CONE WITH ACTUAL VALUES  
 IN SIERRA NEVADA MOUNTAINS AREA

Station Number	USGS Terrain Correction	T.C. = $111.9\Delta h$ $\sin \alpha$	$\Delta$ T.C. USGS Empirical T.C.
240	3	1	2
241	8	10	2
242	7	4	3
243	5	5	0
244	6	4	2
245	5	9	4
246	6	7	1
247	9	12	3
248	1	0	1
249	1	0	1
250	2	1	1
251	4	2	2
252	3	3	0
253	1	0	1
254	1	1	0
255	3	3	0
256	1	0	1
257			
258			
259			
260			

TABLE 8  
DIFFERENCE IN PREDICTED AND OBSERVED  
TERRAIN CORRECTION VALUES IN SIERRA NEVADA MOUNTAIN AREA

Diff.	Cases	Percent	$\Sigma$ Cases	$\Sigma$ Percentage
0-2 mgal	140	54	140	54
3-5	63	25	203	79
6-8	28	11	231	90
9-11	15	6	246	96
12-14	6	2.4	252	98
15-17	3	1.3	255	99
18	1	0.3	256	100

the total sample there are just as many positive as negative differences in values. Table 9 also shows that although the magnitude of the error increases with the size of the correction the percentage error decreases and has an average value of about 22 percent. As most of the terrain correction values (226 or 88 percent of the total sample) are in the range from 0 to 25 mgal where the average error is about 30 percent, the absolute error most of the time is less than 4 mgal.

If these same data are examined on a statistical basis in terms of the topographic relations associated with each station, the results are as shown in Table 10. As might be expected, there is a close correlation with implied relief and its form, and as shown the standard deviation for each group changes in a parallel manner with that for the average deviation for each group.

## 7.2 Test of the Graphical Method for Determining the Terrain Correction

To test the graphical method for determining the terrain correction from the theoretical curves of Figure 10, the terrain corrections for the 1243 stations utilized in the preliminary study and whose general locations are shown in Figure 5 were used. The input values, as in the Sierra Nevada Mountain test area, were the maximum slope

TABLE 9

DEVIATIONS OF PREDICTED VALUES OF TERRAIN CORRECTION FROM  
OBSERVED VALUES AS A FUNCTION OF CORRECTION MAGNITUDE

Magnitude	Zero Diff. Cases	(+) Values Cases	Ave.	(-) Values Cases	Ave.	Ave. Diff.	% Diff.
0-5 mgal	12	6	2.3 mgal	25	1.4 mgal	1.0	40
5-10	8	25	2.7	24	2.5	2.3	31
10-15	4	33	4.8	25	3.4	3.9	31
15-20	1	18	6.1	15	3.3	4.6	26
20-25	4	18	5.2	4	2.5	4.0	18
25-30	0	5	6.6	5	6.6	6.6	24
30-35	2	3	4.3	6	5.2	4.4	14
35-40	0	3	9.6	1	4.0	8.2	22
40-50	1	1	5.0	5	8.0	6.4	14
50-80	0	0		2	2.5	2.5	4
Total	32	112		112	Group Ave.	4.4	22

TABLE 10

## STATISTICAL SUMMARY OF FRESNO TEST AREA

Topo Type	Terrain Correction Range						Topo Type Total	Total
	1-4 Mgals	5-10	11-20	21-30	31-40	41 & Over		
	Number or Stations							
Nodes	0	0	8	4	0	1	13	
Peaks	0	0	1	10	7	7	25	
Ridges	1	7	5	5	1	0	19	254
Slopes	29	34	35	8	1	1	108	
Valleys	2	25	50	10	2	0	89	

TABLE 10. (Continued) STATISTICAL SUMMARY OF FRESNO TEST AREA

Topo Types	Terrain Correction Range						Topo Type Average	Bias
	1-4 Mgals	5-10	11-20	21-30	31-40	41 & Over		
			Average $\Delta$ TC					
Nodes	--	--	+3.00	+3.00	--	--	+2.77	
Peaks	--	--	--	+1.40	+0.14	-3.58	-0.04	
Ridges	--	+1.57	+2.40	-1.00	--	--	+0.47	+0.784
Slopes	-0.69	+0.47	-0.08	+1.50	--	--	-0.09	
Valleys	-1.00	-0.20	+1.74	+6.70	+9.00	--	+1.85	



TABLE 10. (Continued) STATISTICAL SUMMARY OF FRESNO TEST AREA

Topo Types	Terrain Correction Range						Topo Type Std. Dev.	Standard Deviation
	1-4 Mgals	5-10	11-20	21-30	31-40	41 & Over		
			Standard Deviation					
Nodes	--	--	<u>+7.02</u>	<u>+10.77</u>	--	--	<u>+8.12</u>	
Peaks	--	--	--	<u>+ 4.29</u>	<u>+5.33</u>	<u>+6.77</u>	<u>+5.60</u>	
Ridges	--	<u>+3.09</u>	<u>+5.97</u>	<u>+ 8.52</u>	--	--	<u>+6.02</u>	<u>+4.88</u>
Slopes	<u>+1.26</u>	<u>+2.92</u>	<u>+3.66</u>	<u>+ 4.27</u>	--	--	<u>+3.23</u>	
Valleys	<u>+1.00</u>	<u>+2.13</u>	<u>+5.56</u>	<u>+ 8.42</u>	<u>+10.30</u>	--	<u>+5.39</u>	

out to 1 or 2 km from the station and the elevation difference between the station and the surrounding 15' x 15' size area terrain.

The results of this test are summarized in Table 11, and as seen the standard deviation varies from 2 to 7 mgals for the various areas. The average value for all areas being 5 mgals which agrees closely with that obtained for the Sierra Nevada Mountain area and further verifies the applicability of the method.

### 7.3 Conclusions Regarding Test Evaluations

The test results obtained are believed to be representative and significant since many of the stations were located in extremely rugged terrain. In the Sierra Nevada Mountain test area using Equation (8) for determining the terrain correction only 36 of the 254 stations were located in the lower foothill area of the Sierra Nevada Mountains at elevations below 2000 feet. The rest extended up to the crest of Mount Whitney with an elevation of 14,494 ft. The area involved was large and embraced, as brought out in Table 10, a variety of topographic forms ranging from isolated mountain peaks and glacially formed ridges and cirques to deeply incised canyons. In fact, it would have been difficult to find an area with more rugged topography and with such a gradation of relief such as is

TABLE 11

STATISTICAL EVALUATION OF RESIDUALS  
 USING THEORETICAL CURVES FOR DETERMINING THE  
 TERRAIN CORRECTION IN DIFFERENT AREAS IN CALIFORNIA

AMS Sheet	Map Area	n	$\Sigma\Delta^2$	$\Sigma\Delta^2/n$	$\sigma = \sqrt{\frac{\Sigma\Delta^2}{n}}$
L.A. & Long Beach	I	86	1436	17	4
Bakersfield	II	135	6487	60	8
Santa Maria	III	83	1025	12	3
Santa Maria	IV	41	199	3	2
Santa Rosa	V	111	1223	11	3
Redding-Eureka	VI	193	2603	14	4
San Luis Obispo Santa Cruz	VII	291	5554	20	5
Redding-Ukiah	VIII	108	4124	38	6
Chico	IX	38	895	24	5
Ukiah-Redding	X	157	7664	49	7
Total		1243	31210	25	5

n = number of stations

$\Delta$  = observed correction-predicted (curve) correction

associated with the Sierra Nevada Mountains and its western foothills region.

That under such extreme conditions and using a map on a scale of 1:250,000 with 200 contours an average standard deviation of only 4.8 mgals between predicted and observed values that ranged from 1 to 74 mgals suggests that under less rigorous terrain conditions the agreement would be even better. That this would be the case is clear from the data of Table 9. That the theoretical curves of Figure 11 could be used with a similar degree of agreement in areas ranging from the coastline up into portions of the Klamath Mountains and northern Sierras further verifies the applicability of the method.

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