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AN INVESTIGATION OF TWO MODELS OF THE DOPPLER RECORDINGS
ASSOCIATED WITH EARTHQUAKE RAYLEIGH WAVES

A THESIS SUBMITTED TO THE GRADUATE DIVISION OF THE
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ABSTRACT

Doppler shifts of stable radio signals reflected from the ionosphere have been associated with the passing of Rayleigh waves over the surface of the earth. Two proposed models of this phenomenon are examined. A multilayer approach used in the study of normal modes cannot be used to model the Doppler motion because it is unable to account for the time difference between the Rayleigh and Doppler motions. An acoustic propagation model is supported by three forms of evidence: a) the observed amplitude of the Doppler shift is comparable to calculated shifts based on ground motion forcing an acoustic wave upward through the atmosphere b) the phase velocity calculated from two closely spaced Doppler records compare favorably with established Rayleigh phase velocities and c) the time difference between ground and Doppler motions for certain frequency components is comparable to the travel time of an acoustic wave between the two points. However, waves of periods longer than about 80 seconds show a definite increase in travel time with increasing period. This data is incompatible with the acoustic propagation model. An attempt to explain this effect by a modification of the acoustic model considering the effects of attenuation on acoustic velocity was unsuccessful.

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INTRODUCTION

The Radioscience Laboratory at the University of Hawaii, Manoa has been since 1964 monitoring the Doppler shift of a probing radio wave as it reflects off the ionosphere. The Doppler Sounder technique was developed and described by Watts and Davies (1960).

Several examples of disturbances of the ionosphere associated with the passing of Rayleigh waves of an earthquake have been recorded at the Radioscience Lab and have been presented in Yuen et al. (1969) and Weaver et al. (1970).

This thesis examines two models of this phenomena. One is a multi-layer model developed in the study of normal modes. It is concluded that this model, as it has been developed, is unable to describe the Doppler motion. The other model is an acoustic propagation that has been proposed and examined in the work mentioned above. Three forms of evidence, the first two original with this thesis, are presented in support of this model. They are:

- 1) The observed amplitude of the Doppler shift is close to a calculated shift based on the amplitude of ground motion, an appropriate model of the atmosphere and the assumption that the energy is carried upward by a plane acoustic wave.

- 2) A seismic method to find phase velocity from records of two nearby stations was applied to two Doppler recordings of the same earthquake. There is reasonable agreement with established values of Rayleigh wave phase velocity.

- 3) The measured travel time of portions of the acoustic wave from earth to the altitude of the Doppler reflection is comparable to the travel time calculated assuming a velocity structure of the atmosphere.

However, the travel time for other portions of the wave are delayed with respect to a predicted travel time based on the same velocity structure. This effect has not yet been explained and prevents acceptance of a simple acoustic model.

The next section will provide a brief background of the Doppler recording system at the Radioscience Lab, where all the ionospheric data used in this study was gathered. Chapter three will explain the study done of the multilayer model approach. The next chapter will detail the studies of the acoustic model and will be followed by conclusions.

DESCRIPTION OF STATION AND DOPPLER RECORDS

The Radioscience Laboratory at the University of Hawaii, Manoa (157.49 W, 21.18 N) on Oahu operates receiving and recording systems for 5 and 10 MHz. The transmitted signals are the highly stable standard time signals from WWVH, the National Bureau of Standards field station at Puunene, Maui (156.5 W, 20.15 N). WWVH is presently on Kauai, but at the time the data used in this study was collected it was on Maui. The systems are in continuous operation. Similar 5 and 10 MHz systems have been sporadically in operation at Hana, Maui (156.0 W, 20.8 N) and Kona, Hawaii (155.95 W, 19.6 N).

The receiving stations effectively monitor reflection points in the ionosphere midway between it and the transmitter. Figure 1 is a map showing WWVH transmitter, the Manoa and Kona stations and their midpoints.

The data used in this report are two 10 MHz records from the Manoa station and one 10 MHz record from the Kona station. This data is associated with two earthquakes, one on May 16, 1968 near Hachinohe, Japan and another on August 11, 1969 in the vicinity of the Kuril Islands. Reports have been previously published by Yuen, Weaver, Suzuki, and Furumoto (1969) for the Hachinohe event and by Weaver, Yuen, Prolss and Furumoto (1970) for the Kuril Islands event.

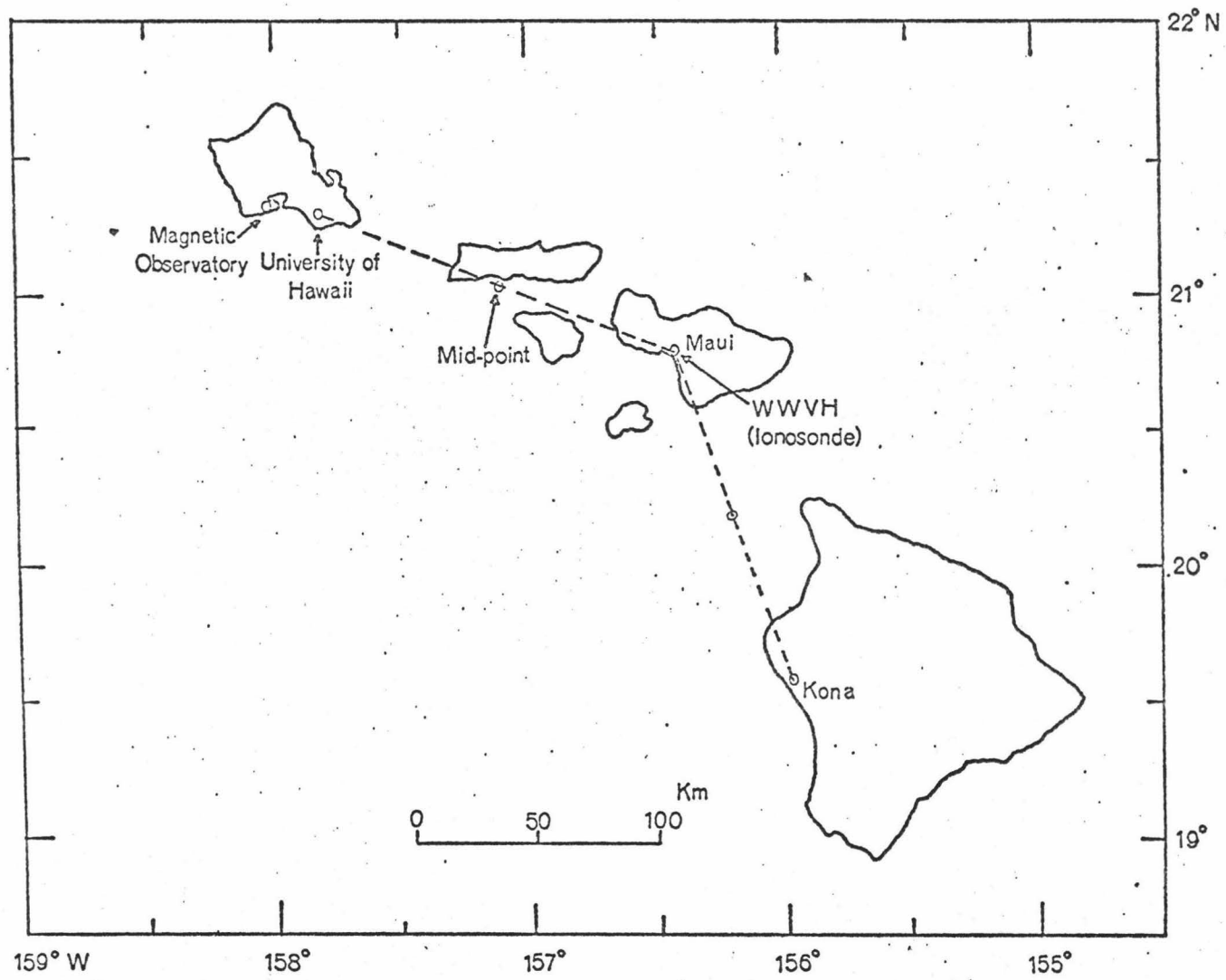


Figure 1

MULTILAYER NORMAL MODE MODEL

There exist models of the atmosphere (Press and Harkrider, 1962; Pfeffer and Zarichny, 1962; Midgeley and Liemohn, 1966; Francis, 1973) and the solid earth (Harkrider, 1964a) which are based on a multilayer method developed by Haskell (1953). This method starts with the earth (or atmosphere) modeled as a stack of horizontal, homogeneous layers. Each layer has its appropriate equation of motion from which perturbation variables are derived. These variables include, for the solid earth case, horizontal and vertical displacement and stress. For the atmospheric model, the variables are the vertical velocity and pressure. At each of the interfaces, the variables are assumed to be continuous. These boundary conditions enable the solution of the equations of motion of one layer to be carried to the next layer. In this way, an analytic solution for the entire structure is found. Matrix algebra and the use of computers are practically necessary to carry out the calculations for a realistic model.

The Doppler group velocity was calculated by dividing the horizontal distance from the earthquake epicenter to the Doppler reflection point by the travel time (difference between the earthquake origin and the first motion of the Doppler arrival). Results revealed velocities of about 2.5 km/sec. This is nowhere near group velocities of any earth or atmospheric model and is better interpreted as the result of energy travel over two paths of different velocities. The combination of paths that fits the data is a surface wave on the earth which generates an almost vertically incident acoustic wave from the ground to the height of reflection of the radio wave (Yuen et al., 1969).

The physical reason for this split in the path of energy is that in

any propagation which involves the solid earth and atmosphere, the solid earth dominates the carrying of energy horizontally away from the source. Harkrider and Flinn (1970) developed a model which includes earth and atmospheric layers (with an intermediate oceanic layer) in their analysis of Rayleigh waves generated by atmospheric explosions. The solutions for group and phase velocity for their model are nearly equal to the solutions for the solid earth only. That this is so is not surprising since models that have been used in comparing calculated and observed surface wave phase and group velocities do not include any atmosphere (Harkrider, 1970) implying that the atmosphere on the real earth has a negligible effect on the wave motion in the solid earth. Apparently, the velocities of the atmosphere and earth are so dissimilar for the periods of motion under study that the structures do not couple effectively. Particle restoring forces in the atmosphere cannot respond to the fast earth motion to build a mode that pervades the entire earth-atmosphere structure. What little energy that does escape into the atmosphere is carried vertically upward into regions where it is dispersed by the attenuation effects of viscosity and heat conduction.

No matter what the physical reason, the breakup of the path of energy into distinct earth and atmospheric portions renders these multilayer normal mode studies inapplicable to the description of the Doppler motion. Early in the mathematical development of the multilayer models, solutions are assumed by the method of separation of variables. This has the effect of removing the frequency of particle motion from dependence on the vertical coordinate. That is, if the motion at a point on the earth's surface is at a particular frequency then the particle motion at all points vertically above and below it will have the same

frequency. Clearly, this mathematical setup cannot account for the time delay between the Doppler and Rayleigh motion.

The Doppler motion is more appropriately viewed as a continual release of energy from the earth's surface that travels upward through the atmosphere to be dissipated in its upper regions. Before it is entirely dissipated, it produces the motions which are recorded by the Doppler system.

ACOUSTIC PROPAGATION MODEL

A more appropriate model for the Doppler phenomena eliminates the earthquake as the source of the motion and substitutes for it the passing Rayleigh wave. In other words, an acoustic propagation model similar to the one described in Yuen et al. (1969).

Chang (1969) studied conditions for the validity of using rays to model acoustic waves in the atmosphere. The conditions depend on the frequency and propagation angle from vertical of the wave and its elevation in the atmosphere. In graphs he has prepared for waves of three minute period, an acoustic ray model is valid at all heights in an average atmosphere for propagation angles less than about 20 degrees. The Doppler motion studied has component periods all of less than three minutes and, as will be presently shown, propagation angles of less than 15 degrees. Thus this preliminary examination seems to justify the use of an acoustic propagation model.

The energy transfer from earth to atmosphere is viewed in the following manner. As the Rayleigh wave passes over the earth's surface, it continually perturbs the air above it. Considering the velocity of sound in air and of the Rayleigh wave, a plane wave is generated with a launch angle, i , of

$$i = \sin^{-1} v/c$$

where v is the acoustic velocity of air at ground level and c is the Rayleigh phase velocity. Under normal conditions i is about five degrees from vertical.

As the acoustic wave travels through the atmosphere it is continually refracted due to the changes in acoustic velocity. For instance, a plane wave with a five degree launch angle results, in an average atmosphere,

in a thirteen degree angle of incidence at 300 kilometers elevation, which is approximately the reflection height for a 10 MHz signal. Because there is a slight horizontal component of velocity, the launch point on the earth's surface is not directly below the point in the ionosphere which is monitored by the Doppler method. Again for a 10 MHz signal in an average atmosphere, this point is about 50 kilometers nearer the epicenter than the midpoint of station and transmitter.

The remainder of this thesis presents studies based on the assumption that the motion of the ionosphere is caused by the interaction of electrons with an acoustic wave generated by the vertical motion of Rayleigh waves.

AMPLITUDE

The amplitude of the compressional wave in the air just above the earth's surface is set equal to the ground vertical motion. The amplitude of this wave of electrically neutral particles grows primarily as the density of the atmosphere decreases with increased elevation. The ratio of wave amplitude at reflection height to ground amplitude was determined by a method described in Chang (1969, pp. 86-100) and Baker and Cotten (1971). It is a multilayer method which, by assuming a velocity structure of the atmosphere, calculates reflection and transmission coefficients at the interfaces of each layer.

The neutral wave interacts with the charged particles in the ionosphere. But the electron motion is affected by the magnetic field in such a way that the electrons follow the component of the neutral motion along the magnetic field (Baker and Cotten, 1971; Chang, 1969, p. 103). Thus, the electron velocity is

$$\vec{v}_e = A_n (\cos \theta) \hat{e}_\theta \quad (1)$$

where \hat{e}_θ is a unit vector in the direction of the magnetic field, θ is the angle between the direction of propagation of the neutral wave and the magnetic field, and A_n is the velocity amplitude of the neutral wave. The direction of propagation is dependent on the azimuth of the incoming wave and the amount of change in the angle of incidence of the acoustic wave due to refraction along its path. As was mentioned previously, if a five degree launch angle is assumed, the angle of incidence increases to about thirteen degrees at 300 kilometers, the approximate reflection height of a 10 MHz signal.

The vertical component of electron velocity is

$$w_e = \vec{v}_e \cdot \hat{e}_z = A_n \cos \theta \cos \bar{I} \quad (2)$$

where \bar{I} is the complement of the inclination of the magnetic field, which was assumed to be equal to its value at sea level.

From this point, following directly the form of Baker and Cotten (1971), the Doppler shift (in Hz) of the radio wave can be calculated by

$$\Delta f = \frac{-\kappa}{fc \cos \phi} \left\{ \int_0^{h_r} \frac{1}{\mu(z)} \cos \theta \cos \bar{I} N(z) \frac{d\omega}{dz} dz + \int_0^{h_r} \frac{1}{\mu(z)} \cos \theta \cos \bar{I} W \frac{\partial N}{\partial z} dz \right\} \quad (3)$$

where W is the neutral particle velocity

N is the electron number density (km^{-3})

$$\kappa = 8.05 \times 10^{-20} \text{ MHz}^2 \text{ km}^3$$

$$c = 3 \times 10^5 \text{ km/sec}$$

f is the frequency of the radio wave (MHz)

ϕ is the angle of incidence of the radio wave ($^\circ$)

$\mu = (1 - \frac{\kappa N}{f^2})$ the phase refractive index

and h_r is the reflection height of f

Records from the May 16, 1968 earthquake were studied. An atmospheric model chosen from the 1965 COSPAR International Reference Atmosphere (CIRA) appropriate for that date and time of day was used. The electron density profile was derived from ionograms from the National Bureau of Standards ionosonde on Maui.

Equation 3 was integrated numerically. Intervals of one kilometer were taken until near the reflection height where intervals of 0.001 kilometer were used. As expected, most of the frequency shift occurred near the reflection height where μ is small.

The peak to peak Doppler shift resulting from a ground vertical velocity amplitude of 1 mm/sec was found to be 5.7 Hz.

Long period seismograms from the HON station at Ewa Beach, Oahu were available. The vertical component was not in a readable condition. From the east-west component trace, peak to peak amplitude for the 100 second period was about 90 mm. Considering the azimuth of the incoming Rayleigh wave to be about 36 degrees from an east-west direction, a horizontal amplitude of $90/\cos 36 = 111$ mm was derived. Magnification at a 100 second period for this instrument is reported to be 60 in Brune and King (1967). Thus, the horizontal ground displacement amplitude was $111/60 = 1.8$ mm. Harkrider (1970) calculated the ellipticity (the ratio of horizontal to vertical amplitudes) for a 100 second Rayleigh wave over an oceanic path to be 0.697. This indicates that the vertical displacement amplitude of the Rayleigh wave was $1.8/.697 = 2.6$ mm. The vertical velocity amplitude is thus $2.6 \times 2 / 100 = 0.16$ mm/sec.

Therefore, based on these calculations the predicted Doppler shift for a 100 second component on May 16, 1968 is $0.16 \times 5.7 = 0.9$ Hz. The actual Doppler shift for that period was 1.1 Hz.

There are several factors which involve uncertainties and possible error in the calculated shift. For example, the atmospheric model is at best a very good guess based on past observations. Also, the numerical integration is an approximation. Considering these uncertainties that were involved in the calculations, this comparison is good.

PHASE VELOCITY

Associated with the arrival of the Rayleigh waves of the August 11, 1969 earthquake were two 10 MHz Doppler records from Manoa and Kona (Weaver et al. 1970).

Seismograms from two closely spaced stations can be used to find the phase velocity of components of a wave train (Nafe and Brune, 1960). Crests of the wave motion can then be matched. Phase velocity of the frequency associated with each crest is then simply

$$c = \Delta x / \Delta t$$

where Δx is the distance between stations and Δt is the difference in arrival time of the crest at the two stations.

This analysis was carried out on the Doppler records. The original Doppler records were digitized at 0.01 inch intervals (corresponding to about 6.8 seconds on the time scale) and smoothed by eye. Peaks and troughs were then matched. Δx was taken to be the difference in distance from the epicenter to the midpoint of the path between each station and the WWVH transmitter. Δx is more properly the difference in launch points. But considering the epicentral distance, this distance is practically equal to the distance between midpoints. In this study, Δx was 134 km.

The values of phase velocity which resulted from this study are listed in Table 1. Figure 2 plots these points with established values of Rayleigh wave phase velocities reported in Oliver (1962) for periods greater than 150 seconds and Kuo et al. (1962) for periods less than 150 seconds. The calculated values lie slightly but consistently above the line representing observed phase velocities. A possible reason for this is discussed in the next paragraph.

TABLE 1
Doppler Phase Velocity

Period (sec)	Δt (sec)	Phase Velocity (km/sec)
74	33.2	4.04
89	30.3	4.42
101	31.0	4.32
130	31.9	4.20
149	29.4	4.56
168	30.6	4.38
189	28.6	4.69

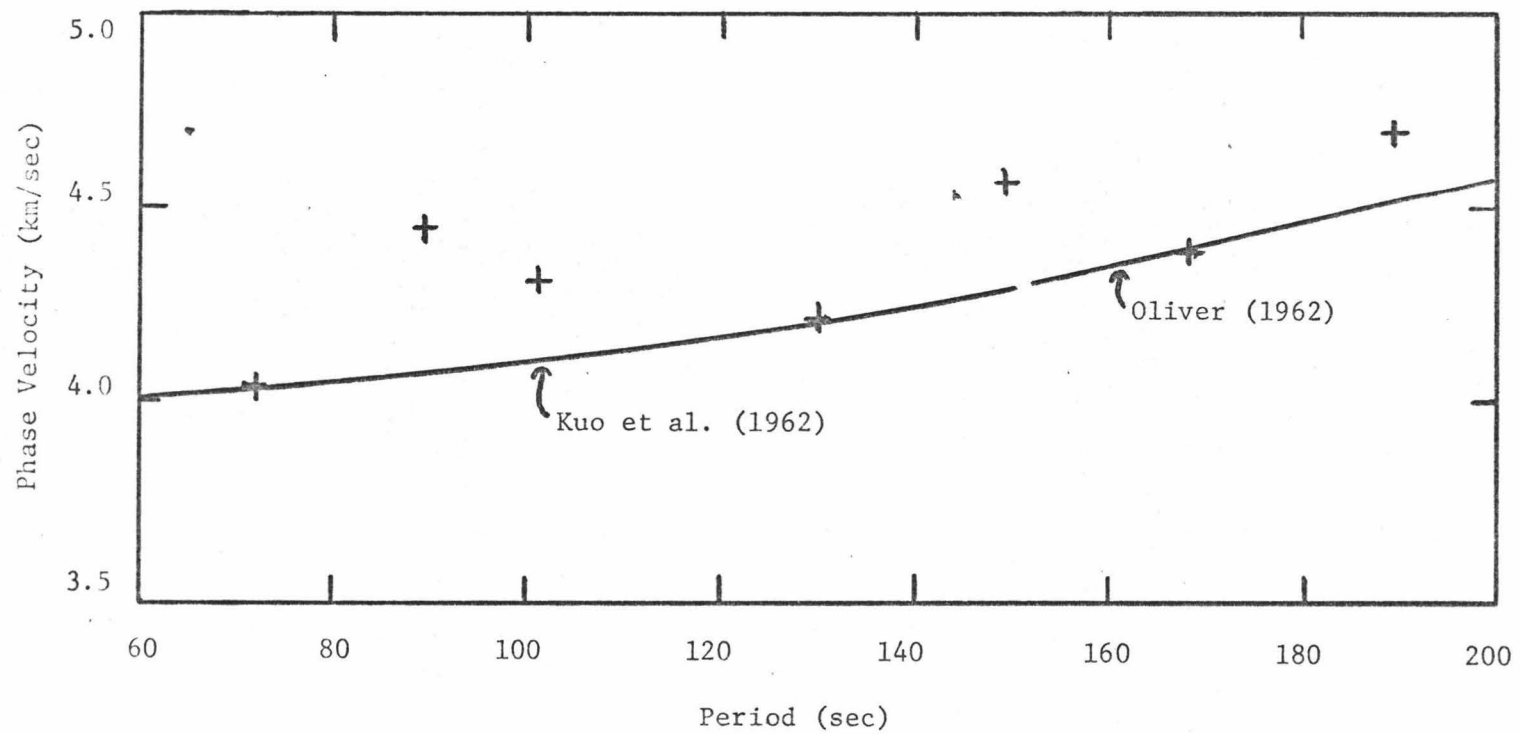


Figure 2. Doppler phase velocity for 11 August 1969

The prime difficulty encountered in this study was the lack of accurate timing on the Doppler records. The timing marks (and the record itself) were fuzzy. Since 0.01 inch on the record corresponds with 6.8 seconds, there is possibly some error in the Δt measurements caused by a slight misalignment of the two records. The work was carried out as carefully as possible, yet, for example, three seconds added to Δt for the 89 second period would lead to a 0.4 km/sec difference in calculated phase velocity. On the other hand, any error in Δt would be equal for all periods and would, if less than a few seconds, lead to approximately the same change in calculated velocity. It thus would have little effect on the trend or slope of the phase velocity curve. Thus a slight realignment would fit the calculated values much better to the established values. But the data does not warrant this review.

As a tool for finding values of Rayleigh wave phase velocities, this method will not be useful until a more accurate timing and recording system is developed. Even then, since the atmosphere is another variable along the path of the wave energy, precise values cannot be expected. One of the assumptions of this study is that the atmospheric effect along both wave paths are equal. For the small spacing between recording points and the short times involved, this is a reasonable assumption, especially considering the present lack of precision in the timing. With improved precision, slight inhomogeneities in the atmosphere would become apparent in the results.

GROUP VELOCITY

The group velocity of various components of a wave train can be found by a peak and trough method described in Ewing and Press (1954). This method was applied to the Doppler traces.

The group velocity was taken to be the distance from epicenter to launch point of the acoustic wave divided by a corrected travel time which represented propagation over the surface of the earth. At first, the correction to the travel time was a subtracted constant for all frequencies and represented the travel time of an acoustic wave from ground to reflection height. The travel time of an acoustic wave will vary with frequency since the launch angle of the wave is dependent on the phase velocity of the Rayleigh wave. However, in the present study, the difference in travel time between the slowest and fastest wave is about one second. Table 2 lists and Figure 3 plots the results of this study of the May 16, 1968 and the August 11, 1969 events compared with a curve that matches compiled group velocities of the Pacific Ocean area (Kuo et al. 1962). The uncorrected travel time is the time difference between earthquake origin to Doppler arrival. The acoustic wave travel time correction was calculated from a CIRA temperature model appropriate for the date and time of day. In Figure 3, this correction (in seconds) is the number in parentheses following the date. The resulting dispersion curve has a steeper slope than previous studies of Rayleigh waves indicate.

Another view of this discrepancy was gained by defining a group delay as the difference between the corrected travel time and a calculated travel time based on group velocities established in the Kuo paper. Table 2 lists the details of the study and Figure 4 plots group delay

TABLE 2

Doppler group velocity and group delay

Period (sec)	Uncorrected Travel Time (sec)	Corrected Travel Time ¹ (sec)	Doppler Group Velocity ² (km/sec)	Theoretical Group Velocity ³ (km/sec)	Theoretical ² Travel Time ² (sec)	Group Delay (sec)
<u>16 May 1968</u>						
42	2137	1477	4.04	3.99	1496	-19
79	2197	1537	3.88	3.86	1547	-10
88	2258	1598	3.74	3.84	1555	43
108	2307	1647	3.63	3.78	1580	67
120	2364	1704	3.50	3.74	1596	108
162	2434	1774	3.37	3.64	1640	134
170	2518	1858	3.21	3.62	1649	209
<u>11 August 1969</u>						
54	2113	1433	3.92	3.95	1423	10
90	2165	1485	3.79	3.84	1464	21
114	2240	1560	3.60	3.76	1495	65
136	2302	1622	3.47	3.70	1520	102
170	2378	1698	3.31	3.62 ⁴	1533	145
188	2514	1834	3.06	3.60 ⁴	1562	272

1. Travel time corrections: 16 May-660 sec. 11 August-680 sec.
2. Epicentral distance: 16 May-5971 km. 11 August-5623 km.
3. From Kuo et al. (1962)
4. From Oliver (1962)

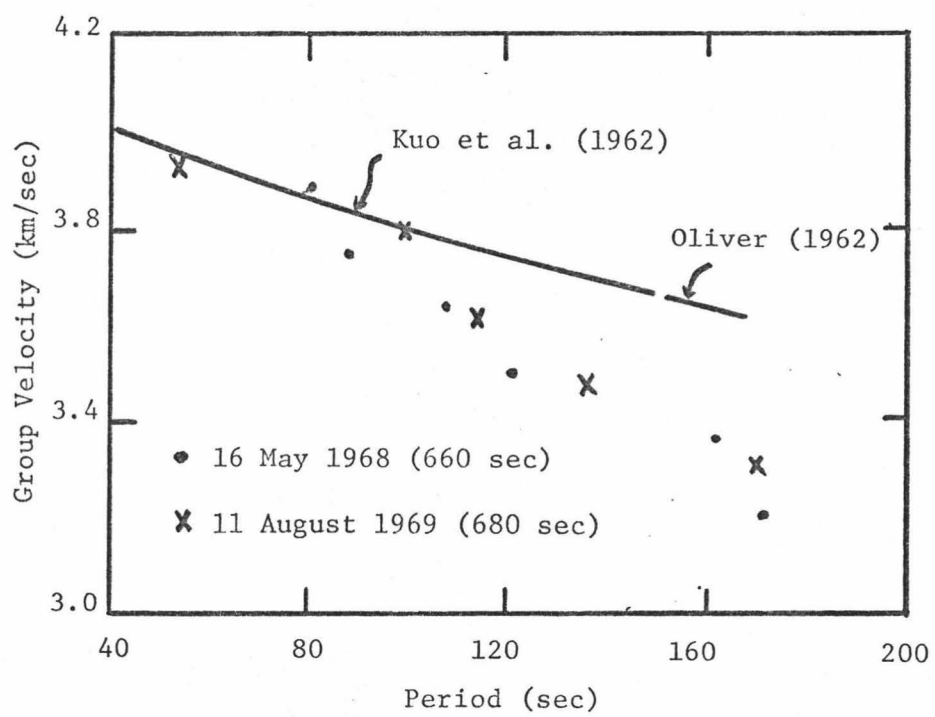


Figure 3. Doppler Group Velocity

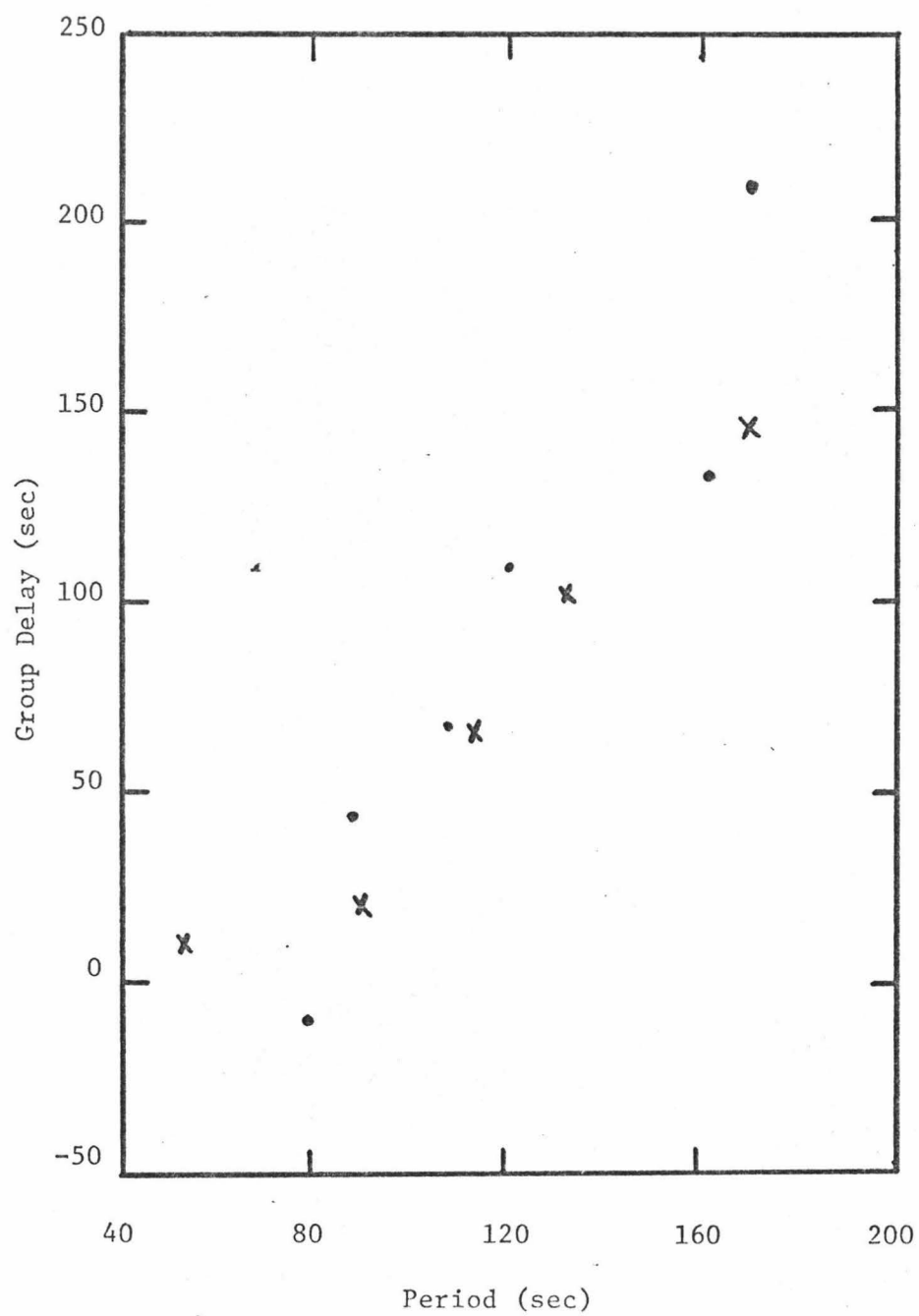


Figure 4. Doppler Group Delay

versus period for the earthquakes studied. The graph indicates an almost linear increase in group delay between periods of 80 to 170 seconds. For shorter periods, group delay is negative in one case and positive in the other. Here, as in the phase velocity study, the lack of accurate timing marks on the Doppler records make time readings subject to error. However, the agreement between the two records, at least between 80 and 170 seconds, supports the magnitude of the delay effect.

The implication of this study so far is that there is some additional dispersion of the Rayleigh wave motion occurring in the atmosphere. This means that a simple acoustic wave propagation model cannot be used to completely describe the Doppler phenomenon associated with earthquake waves.

In an attempt to explain this group delay, a second corrected travel time was tried in connection with a study of the effects of attenuation on acoustic velocity. The velocity of an acoustic wave is independent of its frequency when there is no energy dissipation term in the equation of motion. Upon introducing viscous dissipation, there results an infinite series in the exponent of the solution (Lamb, 1932, p. 647). Appendix A reviews this problem. The terms of this series alternate between real and imaginary. The imaginary terms become the decay factor of the wave amplitude. The real elements become velocity terms. If the viscosity is small, which is the case in the lower atmosphere, only the first order term is retained. This imaginary term becomes the decay factor that is commonly used in basic acoustic texts. However, in the upper atmosphere the coefficient of viscosity becomes increasingly large so that the higher order terms cannot be neglected. Appendix A includes an original method which avoids the expansion and results in an exact

solution of the problem. As shown there, the effect of attenuation is to make the acoustic velocity dependent on frequency. The formula for the velocity of sound becomes

$$c = \frac{Rc_0}{\cos \phi}$$

where $R = (1 + N^2)^{1/4}$

$$\phi = 1/2 \tan^{-1} N$$

$$N = \frac{4}{3} \frac{\nu \omega}{c_0^2}$$

c_0 is the velocity of sound in the absence of viscosity

ν is the kinematic viscosity coefficient

ω is the angular frequency of the acoustic wave

Using the formula for phase velocity, the travel time for different frequencies through a multilayer atmosphere were calculated. This travel time was used to derive an average phase velocity for the entire path. From this, an average group velocity, u , was calculated by

$$u = c + k \, dc/dk$$

where k is the wave number. Figure 5 is a graph of the average phase and group velocities to a height of 304 kilometers for an average atmospheric model from the 1966 U.S. Standard Atmosphere Supplements. There is an increase in phase and group velocity with decreasing period. Calculations were stopped at about 50 seconds because attenuation eliminates from observation waves of higher frequency (Yuen et al. 1969). Fourier power spectra of the August 11, 1969 Doppler record showed very little energy below 60 second period. Indeed, the critical factor of this study is the examination of how much the velocity is affected before the wave amplitude is reduced to an unmeasurable level. Toward the longer periods, velocities asymptotically approach a value that

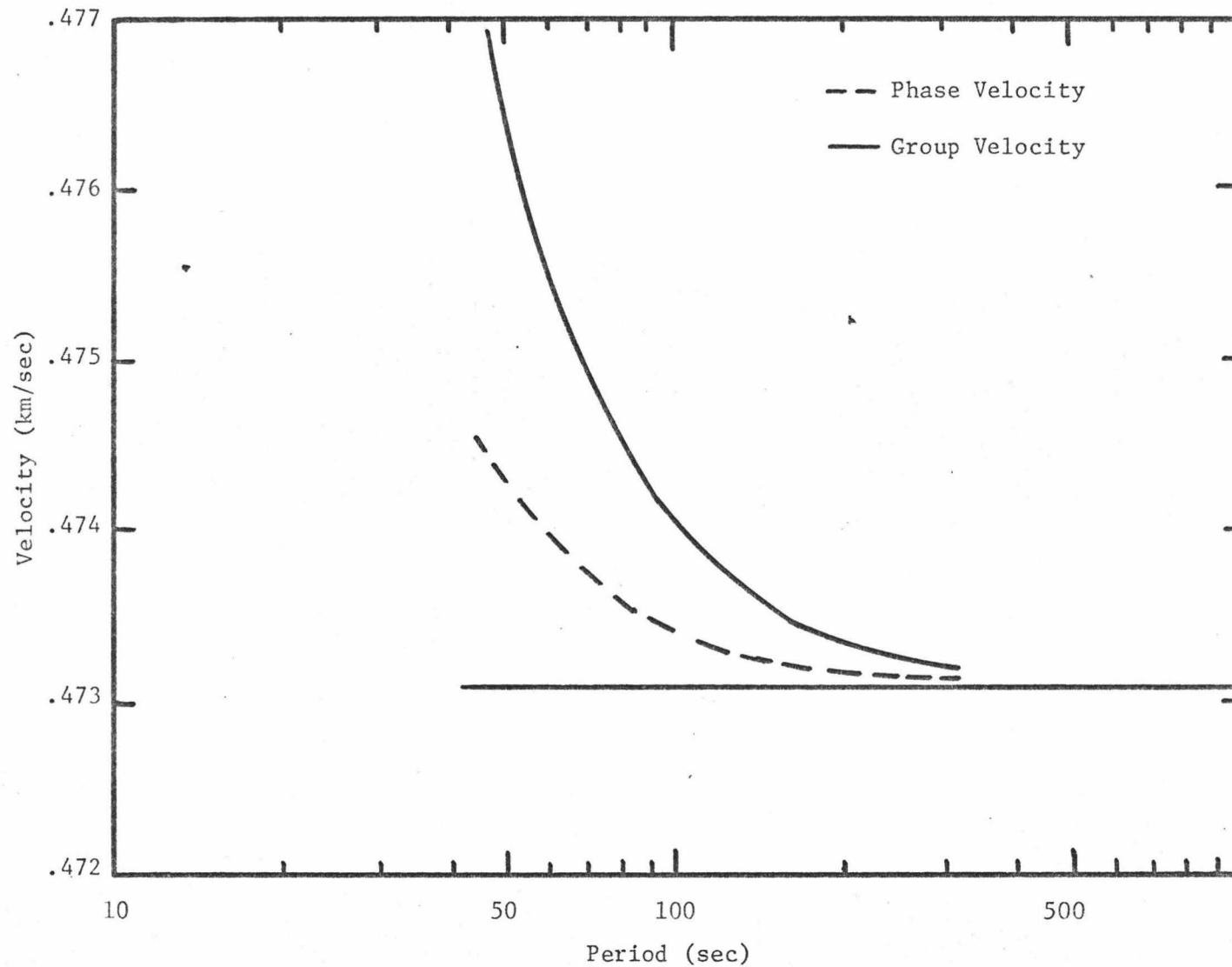


Figure 5. Average phase and group velocities to 304 km including viscosity effects.

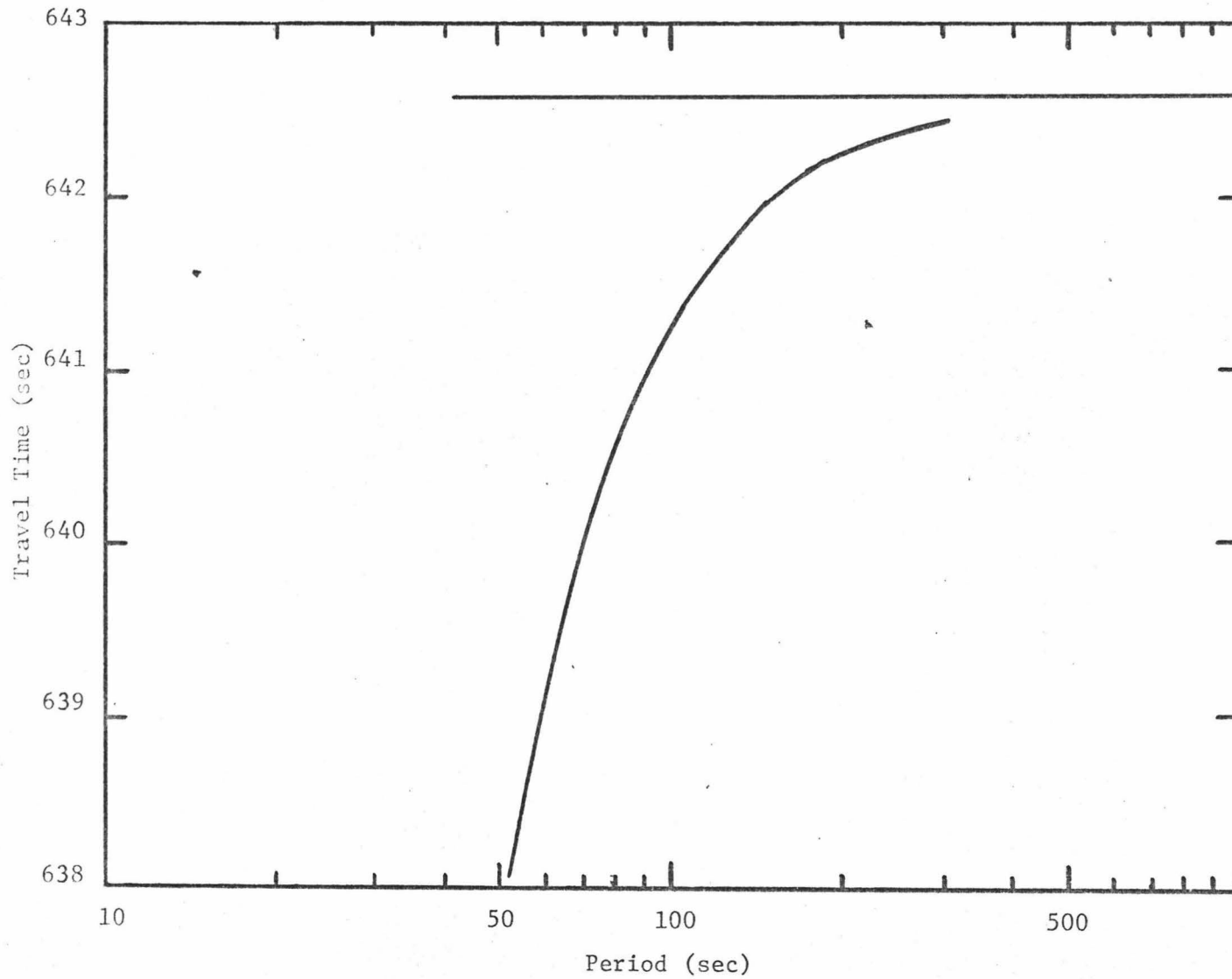


Figure 6. Group travel time to 304 km including viscosity effects

would be the average acoustic velocity if there were no attenuation. This value is represented by the horizontal line at about 0.4731 km/sec. Calculations were stopped, however, at 300 seconds since the assumption for acoustic plane waves is not valid for longer periods (Chang, 1969, p. 83).

In Figure 6, the group velocities of Figure 5 have been converted to travel times. The increase in velocity toward shorter periods is naturally converted to smaller travel times. At longer periods, the time approaches that of an unattenuated wave, represented by the horizontal line at about 642.6 seconds.

This study predicts no positive group delay but instead a group advance (or negative group delay). At a 60 second period, this amounts to about 3.3 seconds. At 90 second period, a 1.6 second advance is predicted. These times are definitely not measurable since they are well within uncertainties of reflection height, timing and atmospheric modeling.

The attenuation effects of heat conduction were not included in this analysis but should be about the same quantitatively as the effect of viscosity (Chang, 1969, pp. 213-216). But even if heat conduction had a similar effect on velocity and the amount of group advance predicted for viscous effects were doubled, it would still not be significant.

In conclusion, this study fails on two accounts to explain the observed group delay. One, no positive delay times are predicted and whatever time change is calculated is too small by at least an order of magnitude to be measurably significant. Two, at long periods, the delay time asymptotically approaches zero whereas the observed delay time becomes very large for long periods. The group delay effect remains

unresolved.

A sidelight of the group velocity study applies to the travel time of the acoustic wave through the atmosphere. Yuen et al. (1969) compare the arrival times of a portion of the Rayleigh wave with a similar portion of the Doppler trace of the May 16, 1968 event. This difference in time was in turn compared with a travel time calculated by assuming an atmospheric temperature model appropriate for that time. The difference of the measured travel time from the calculated one was about half a minute out of a total travel time of about 11 minutes. Considering the uncertainties involved in the atmospheric model and knowledge of the reflection height, it was claimed to be a favorable comparison and was used as evidence for the adoption of an acoustic propagation model.

In light of this group velocity study, the reason why the comparison in Yuen et al. (1969) worked was that the portions of the wave train that were compared included periods (80-90 seconds) that have small group delay. For other portions of the record, the additional dispersion introduced somewhere between the earth's surface and the Doppler recording affects the waveform so much that it is useless to try peak to peak comparison of Rayleigh and Doppler traces.

CONCLUSIONS

The multilayer method used in the study of normal modes is not applicable to the Doppler records associated with Rayleigh waves primarily because it cannot account for the time difference between Rayleigh and Doppler arrival times.

The phase velocity study indicates that the Doppler traces are derivative from the Rayleigh wave motion. The evidence presented there effectively eliminates almost any other mechanism for the motion because it seems highly unlikely that another source would cause wave motion that would produce results that match Rayleigh phase velocities. On the other hand, the phase velocity study reveals little about the atmosphere or mode of energy transmission through it.

The amplitude study reveals more on the assumption of acoustic wave propagation. The comparability of calculated and observed frequency shifts confirms the amplitude growth predicted for a plane wave traveling upward from the earth's surface to reflection height.

The group velocity study adds some evidence for an acoustic propagation model, but mainly detracts from it. The bit of favorable evidence is that the travel time for an acoustic wave from ground to reflection height is approximately matched by the difference in arrival times by at least a portion of the wave train.

However, since acoustic velocity, if attenuation is small, is independent of frequency, the acoustic model cannot explain the group delay summarized in Figure 4. The delay is about twenty percent of the atmospheric travel time for a wave of 170 second period.

What all of this implies cannot be fully ascertained at present. The favorable evidence for an acoustic propagation model from the phase

velocity and amplitude studies plus the partial agreement in travel time seem too formidable to warrant abandoning the model because of the group delay effect. It seems to the author that an acoustic propagation model of the Doppler motion should be retained and that modifications of the model should be made in attempts to explain the group delay.

It is difficult to conceive the Doppler records as a source for more precise seismic data, even if the group delay problem is solved and if more accurate instrumentation is developed. The reason for this is that the effect of the atmosphere at any particular time can only be approximated since it is constantly in motion. This does not rule out, however, their use in studies which do not require such precision.

Nevertheless, the Rayleigh generated Doppler wave extends the field of seismology to the atmosphere in a way that might previously have been unexpected. Furumoto (1970) has already made a seismic study using Doppler data. The Doppler motion is also a new tool in the study of the atmosphere. Earthquakes are a uniquely large energy generator and as such are a potential source of interesting atmospheric data.

APPENDIX A

See Lamb (1932, pp. 646-647) for a detailed derivation of equations 1 to 9.

Assume plane acoustic (compressional) waves propagating in the x direction. Neglecting second order terms in velocity, the equation of motion is

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{4}{3} \nu \frac{\partial^2 u}{\partial x^2} \quad (1)$$

where p is the pressure

u, v, w are the components of particle velocity in the x, y, z directions, respectively

ν is the kinematic coefficient of viscosity

ρ_0 is the ambient density

Introducing c_0 , the velocity of sound in the absence of viscosity, (1) becomes

$$\frac{\partial^2 u}{\partial t^2} = c_0^2 \frac{\partial^2 u}{\partial x^2} + \frac{4}{3} \nu \frac{\partial^3 u}{\partial x^2 \partial t} \quad (2)$$

Assume a solution to (2) as

$$u = a e^{i\omega t + mx} \quad (3)$$

This leads to

$$m^2 (c_0^2 + \frac{4}{3} i\nu\omega) = -\omega^2 \quad (4)$$

$$m = \pm \frac{i\omega}{c_0} \left(1 + \frac{4}{3} i \frac{\nu\omega}{c_0^2}\right)^{-1/2} \quad (5)$$

At this point, Lamb expands the radical term.

$$\left(1 + \frac{4}{3} i \frac{\nu\omega}{c_0^2}\right)^{-1/2} = 1 - \frac{2}{3} \frac{i\nu\omega}{c_0^2} - \frac{1}{2} \left(\frac{\nu\omega}{c_0^2}\right)^2 + \dots \quad (6)$$

Then by neglecting higher order terms of $\nu\omega/c_0^2$ and taking the lower sign, (5) simplifies to

$$m = -\frac{i\omega}{c_0} - \frac{2}{3} \frac{v\omega^2}{c_0^3} \quad (7)$$

Substituting back into the solution (3)

$$\begin{aligned} u &= a e^{-\frac{2}{3} \frac{v\omega^2}{c_0^3} x} e^{i\omega(t - \frac{x}{c_0})} \\ &= a e^{-\frac{x}{\ell_0}} e^{i\omega(t - \frac{x}{c_0})} \end{aligned} \quad (8)$$

where $\ell_0 = \frac{3c_0^3}{2v\omega^2}$

The decay factor, ℓ_0 , is the one used in basic acoustic texts (Rayleigh, 1945, p. 322).

Lamb notes that "the wave velocity is, to the first order of $\frac{v\omega}{c_0^2}$, unaffected by the friction" (p. 647).

But when $\frac{v\omega}{c_0^2}$ becomes larger, as is the case in the upper atmosphere, its higher order terms cannot, naturally, be ignored. Thus, instead of expanding the radical term (6), a different approach is used.

Let

$$N = \frac{4}{3} \frac{v\omega}{c_0^2} \quad (10)$$

then

$$\begin{aligned} \left(1 + i \frac{4}{3} \frac{v\omega}{c_0^2}\right)^{-1/2} &= (1 + iN)^{-1/2} \\ &= [(1 + N^2)^{1/2} e^{i \tan^{-1} N}]^{-1/2} \\ &= (1 + N^2)^{-1/4} e^{-1/2 i \tan^{-1} N} \\ \left(1 + i \frac{4}{3} \frac{v\omega}{c_0^2}\right) &= R^{-1} e^{-i\phi} \end{aligned} \quad (11)$$

where

$$R = (1 + N^2)^{1/4} \quad (12)$$

$$\phi = 1/2 \tan^{-1} N \quad (13)$$

Substituting back into (5) and then into the solution (4),

$$u = a e^{i\omega t \pm \frac{i\omega x}{c_0} R^{-1} (\cos \phi - i \sin \phi)} \quad (14)$$

By choosing the lower sign

$$\begin{aligned} u &= a e^{-\frac{\omega x \sin \phi}{R c_0}} e^{i\omega(t - \frac{\cos \phi}{R c_0} x)} \\ &= a e^{-\frac{x}{\ell}} e^{i\omega(t - \frac{x}{c})} \end{aligned} \quad (15)$$

where

$$\begin{aligned} \ell &= \frac{c_0 R}{\omega \sin \phi} \\ c &= c_0 \frac{R}{\cos \phi} \end{aligned}$$

The decay factor, ℓ , and acoustic velocity, c , reduce to ℓ_0 and c_0 when N is small.

$$\ell = \frac{R c_0}{\omega \sin \phi} \rightarrow \frac{1 \cdot c_0}{\omega \phi} \rightarrow \frac{c_0}{\omega \frac{N}{2}} = \frac{3c_0^3}{2\sqrt{\omega^2}} = \ell_0$$

$$c = \frac{c_0 R}{\cos \phi} \rightarrow \frac{c_0 \cdot 1}{1} = c_0$$

REFERENCES

- Baker, D.M. and D.E. Cotten, Interpretation of High-Frequency Doppler Observations of Waves from Nuclear and Natural Sources, Jour. Geophys. Res., 76, 1802-1810, 1971.
- Brune, J.N. and C.Y. King, Excitation of Mantle Rayleigh Waves of Period 100 Seconds as a Function of Magnitude, Bull. Seism. Soc. Am., 57, 1355-1365, 1967.
- CIRA, COSPAR International Reference Atmosphere 1965, Compiled by members of Working Group IV of the Committee on Space Research, North-Holland Publishing Company, Amsterdam, 1965.
- Chang, N.J.F., Acoustic Waves in the Ionosphere and Their Effects on High Frequency Radio Propagation, Ph.D. Thesis, Department of Electrical Engineering, University of Colorado, 1969.
- Ewing, M. and F. Press, An Investigation of Mantle Rayleigh Waves, Bull. Seism. Soc. Am., 44, 127-147, 1954.
- Francis, S.H., Acoustic-gravity Modes and Large Scale Traveling Ionospheric Disturbances of a Realistic Dissipative Atmosphere, Jour. Geophys. Res., 78, 2278-2301, 1973.
- Furumoto, A.S., Ionospheric Recordings of Rayleigh Waves for Estimating Source Mechanisms, Tsunamis in the Pacific Ocean, ed. Wm. M Adams, East-West Center Press, Honolulu, 1970.
- Harkrider, D.G., Surface Waves in Multilayer Elastic Media, Part I. Rayleigh and Love Waves from Buried Sources in a Multilayered Elastic Half-space, Bull. Seism. Soc. Am., 54, 627-679, 1964a.
- Harkrider, D.G., Theoretical and Observed Acoustic-Gravity Waves from Explosive Sources in the Atmosphere, Jour. Geophys. Res., 69, 5295-5321, 1964b.
- Harkrider, D.G., Surface Waves in Multilayered Elastic Media, Part II. Higher Mode Spectra and Spectral Ratios From Point Sources in Plane Layered Earth Models, Bull. Seism. Soc. Am., 60, 1937-1987, 1970.
- Harkrider, D.G. and E.A. Flinn, Effect of Crustal Structure on Rayleigh Waves Generated by Atmospheric Explosions, Rev. of Geophys. and Space Phys., 8, 501-516, 1970.
- Haskell, N.A., The Dispersion of Surface Waves on Multilayered Media, Bull. Seism. Soc. Am., 43, 17-34, 1953.
- Kuo, J., J.N. Brune and M. Major, Rayleigh Wave Dispersion in the Pacific Ocean for the Period Range 20 to 140 Seconds, Bull. Seism. Soc. Am., 52, 333-357, 1962.

- Lamb, H., Hydrodynamics, Dover, New York, 738 p., 1932.
- Midgley, J.E. and H.B. Liemohn, Gravity Waves in a Realistic Atmosphere, Jour. Geophys. Res., 71, 3729-3748, 1966.
- Nafe, J.E. and J.N. Brune, Observations of Phase Velocity for Rayleigh Waves in the Period Range 100 to 400 Seconds, Bull. Seism. Soc. Am., 50, 427-439, 1960.
- Oliver, J., A Summary of Observed Seismic Surface Wave Dispersion, Bull. Seism. Soc. Am., 52, 81-86, 1962.
- Press, F. and D.G. Harkrider, Propagation of Acoustic-Gravity Waves in the Atmosphere, Jour. Geophys. Res., 67, 3889-3908, 1962.
- Pfeffer, R.L. and J. Zarichny, Acoustic-Gravity Wave Propagation From Nuclear Explosions in the Earth's Atmosphere, Jour. Atmos. Sci., 19, 256-263, 1962.
- Rayleigh, J.W.S., The Theory of Sound, Vol. II, 2nd edition, Dover, New York, 1945.
- U.S. Committee on Extension to the Standard Atmosphere, U.S. Standard Atmosphere Supplements 1966, Government Printing Office, Washington D.C., 1966.
- Watts, J.M. and K. Davies, Rapid Frequency Analysis of Fading Radio Signals, Jour. Geophys. Res., 65, 2295-2301, 1960.
- Weaver P.F., P.C. Yuen, G.W. Prolss and A.S. Furumoto, Acoustic Coupling Into the Ionosphere from Seismic Waves of the Earthquake at Kuril Islands on August 11, 1969, Nature, 226, 1239-1241, 1970.
- Yuen, P.C., P.F. Weaver, R.K. Suzuki and A.S. Furumoto, Continuous Traveling Coupling Between Seismic Waves and the Ionosphere Evident in May 1968 Japan Earthquake Data, Jour. Geophys. Res., 74, 2256-2264, 1969.