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HIGH PRECISION GEODESY  
APPLIED TO CRUSTAL STUDIES

A THESIS SUBMITTED TO THE GRADUATE DIVISION OF THE  
UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT  
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IN GEOLOGY AND GEOPHYSICS

AUGUST 1978

By

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## ABSTRACT

In order to detect any local or regional movement of the Lunar Ranging Observatory on Maui, a geodetic network of laser ranging terminals has been established on the island. The measured distances between these terminals are used to compute the earth-centered space coordinates of the terminals. Annual measurement of the distances and recomputation of the coordinates of the terminals can show any significant changes in the stations. Any change cannot be considered significant unless the change is greater than the predicted measuring error.

A network of measured gravity stations has been established on the islands of Maui and Oahu to aid in the detection of any regional movement. A change in gravity of  $\pm 0.02$  milligals of any of the stations over a period of time could be cause for further investigation of the station elevations.

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## INTRODUCTION

In mid-1976 a NASA funded project entitled "Crustal Deformation" a geodetic-geophysical program in support of Lunar Ranging Experiment (LURE) on Haleakala, Maui was started. The major purpose of the project was to detect any local or regional movement, in order that LURE could derive maximum information on tectonic plate motion. (see Carter, et al., 1977; also Berg and Sutton, 1977.)

Detection of any local or regional motion was to be attempted in different ways. A network of geodetic lines to be measured using electronic distance measuring techniques was established. Ocean tide gauges were established at three locations on the island. First order levelling surveys will be made to tie the tide guages to the LURE observatory on the summit of Haleakala, and to other points of the island. A borehole tiltmeter was to be installed to monitor any local or regional tilt. A seismic network was established to study the effects of earthquakes originating from the island of Hawaii or from the Molokai fracture zone area. Finally, a gravimetric measurement network was established to attempt to observe any free-air or mass distribution differences with time. This thesis will be concerned with the geodetic line network measured in 1977 and the gravity network.

My participation in the project was concentrated in the geodetic and gravity sections. I participated in all phases of the laser surveys, including preliminary laser site surveys, establishment of new laser terminals, operation of laser, and accumulation of atmospheric data at the instrument, reflector, and in the helicopter.



In addition, during the establishment of the gravity networks, I made all of the gravity observations on Maui and Oahu, and wrote all computer programs discussed in this thesis.

## LASER LINE NETWORKS

The network of geodetic lines established on the island of Maui as part of the crustal deformation study is shown in Figure 1. The main terminal points are marked by either U. S. Coast and Geodetic Survey markers or Hawaii Institute of Geophysics (H.I.G.) installed brass markers. Whenever possible the USCGS markers were used. Secondary terminal points are marked by a phillips head screw screwed into a lead insert.

For measurement of the lines a K & E Rangemaster II was used. The Rangemaster II uses a 10 mW He-Ne laser as the light source. Reflectors used were configurations of cube corner prism reflectors. The Rangemaster II used was modified to give an accuracy of  $\pm 3$  mm.

During laser measurements, atmospheric data were taken at each terminal point. Included in this data were barometric pressure, air temperature, and humidity. Barometric pressures were taken with a Negretti and Zambra digital barometer. Dry bulb and wet bulb temperatures were taken with Assman psychrometers.

Whenever possible a helicopter was used to obtain about 40-60 samples of atmospheric data along the line during the measurement of the longer lines. A digiquartz pressure transducer was used to measure barometric pressure. A thermistor and a hygistor were used to measure temperature and humidity respectively. As approximately 8,000 feet was the maximum altitude to which the helicopter could fly along the line during the summer of 1977, atmospheric data could not be obtained in this manner for lines located on the upper two-thirds of Mt. Haleakala.

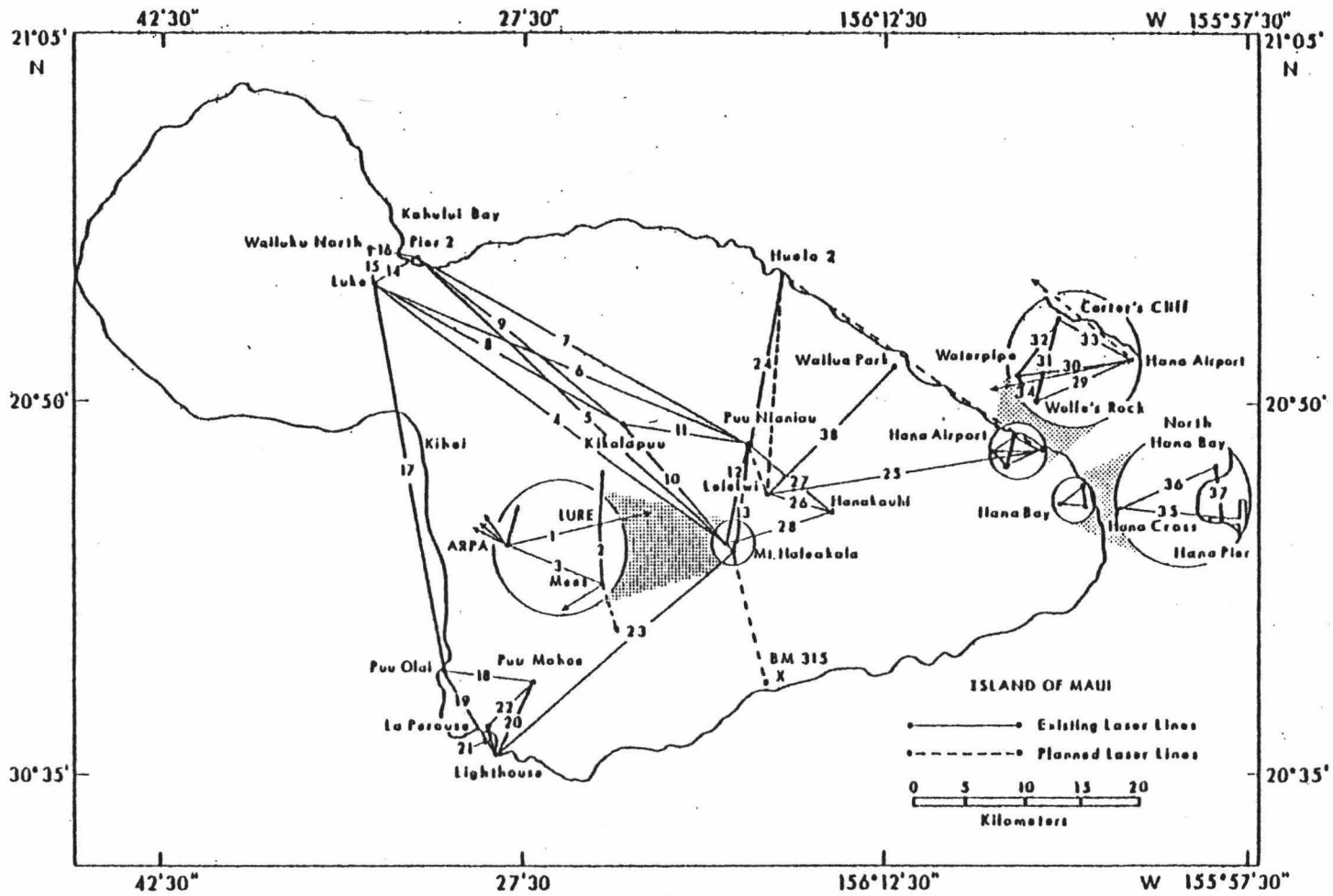


Figure 1. Maui Laser Line Network (From Berg, et al., 1978)

In these cases atmospheric data were obtained from the terminal points and from two intermediate ground stations located as close as possible to the line. All atmospheric data were used to compute a mean refractive index number along the line.

Each distance measurement is taken through a series of reductions to arrive at a marker-to-marker spatial chord distance. These reductions include all of the following:

- (1) laser oscillator frequency correction;
- (2) instrument's linearity correction;
- (3) correction for tripod heights of instrument and reflector above marker elevations;
- (4) eccentricity correction, for when the instrument or reflector were not centered directly over the marker;
- (5) refractive index correction;
- (6) beam curvature correction.

It was found that the total error of a measurement after all of the above reductions were made was  $\pm 3 \text{ mm} \pm 0.2 \times 10^{-6}$  times the distance in kilometers.

After all the above corrections are applied the marker-to-marker spatial chord distance is obtained, which is the point at which the computations in this thesis begin. More information on the instrument modifications and techniques used during measurement can be found by referring to Berg, et al., 1978.

REDUCTION OF LASER LINES TO THE  
HAWAIIAN TRANSVERSE MERCATOR PLANE

After the observed data has been reduced to the marker-to-marker spatial chord distance, the reductions described in this thesis begin. The first reduction to the spatial chord distance is to a distance on the Transverse Mercator plane. This reduction is made to allow data from pre-existing surveys to be compared with the laser ranging surveys, also the conditional adjustment used must be done on the plane.

The thirteen terminal points for which spatial chord distances were reduced to Transverse Mercator plane distances are described below.

1. Luke is marked by a U. H. brass marker located next to a USCGS 1912 marker on a four foot square concrete slab. This site was reoccupied in 1950. Elevation is 93.85 m.
2. ARPA is marked by a US Corps of Engineers marker on a concrete slab 50 meters west of ARPA's westernmost dome. Elevation is 3034.34 m.
3. Pier 2 is marked by a U. H. phillips head screw marker next to USCGS bench mark no. 8 near the Kahului tidal guage. Elevation is 2.28 m.
4. Puu Nianiau is marked by an HGS 1877 VABM marker, which was reoccupied in 1950. Elevation is 2087.90 m.
5. Kikalapuu is marked by a USCGS 1950 VABM marker. Elevation is 755.40 m.

6. Lure is marked by a U. H. brass marker set flushed in concrete in a road approximately 10 meters north of Kolekole. Elevation is 3049.74 m.
7. Mees is marked by a UH brass marker on a concrete slab about 15 meters west of Mees Solar Observatory. Elevation is 3040.99 m.
8. Wailuku north is marked by the signal mast of the HTS 1929 marker, which was reoccupied in 1950. Elevation is 99.70 m.
9. Puu Olai is marked by a UH brass marker on a concrete slab next to an HGS 1879 marker, which was remarked by HTS in 1927 and reoccupied most recently in 1950. Elevation is 110.10 m.
10. Hana Airport is marked by an HTS 1950 VABM marker. Elevation is 16.8 m.
11. Wolfe's Rock is marked by a UH phillips head screw set in a rock approximately 5 meters east of Hana Highway. Elevation is unknown.
12. Hana Waterpipe is marked by a UH phillips head screw in a concrete slab supporting a waterpipe approximately 15 meters east of Hana Highway. Elevation is unknown.
13. Carter's Cliff is marked by a UH phillips head screw set in rock approximately 2 meters from edge of an ocean cliff. Elevation is unknown.

The first step in the reduction process is to find the distance on the reference ellipsoid from the spatial chord distance,  $D_c$ . The following formula was used to find the ellipsoidal arc distance (Wong, 1949; see Laurila, 1960, 1976):

$$d_A = \left[ \frac{12 R^2 M}{12(R+H)(R+K) - M} \right]^{\frac{1}{2}} \quad (1)$$

where  $M = D_c^2 - (H - K)^2$ .  $H$  and  $K$  are the elevations of the terminal points.  $R$  is the earth's radius of curvature in the direction of the line.  $R$  is computed from the following formula:

$$R = \frac{NM}{M \sin^2 A + N \cos^2 A} \quad (2)$$

where  $N$  and  $M$  are the radius of curvature of the meridional arc, and the radius of curvature in the prime vertical respectively.  $N$  and  $M$  are found from the following formulae:

$$N = \frac{a}{w} \quad (3)$$

$$M = \frac{a(1-e^2)}{w^3}$$

where  $a$  is semimajor axis of the reference ellipsoid,  $e^2$  is the eccentricity square of the ellipsoid, and  $w$  is the term:  $(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}$ , where  $\phi$  is the mean latitude of the two terminal points. The reference ellipsoid used in all computations is that of Clarke 1866.

After the ellipsoidal arc distance is found, the next reduction is to find the distance on the Hawaiian Transverse Mercator (HTM) plane. The following formula was used for this reduction:

$$M = d_A \left( 0.9999667 + \frac{(X1^2 + X1 \cdot X2 + X2^2)}{6 \cdot R^2} \right) \quad (4)$$

where 0.9999667 is the scale factor at the central meridian, which is  $156^{\circ}40'00''W$  for Maui, and  $X1$  and  $X2$  are the plane distances from the central meridian of the terminal points. The Fortran computer program REDUC in Appendix A was used to make all of the above reductions. Table 1 shows the reductions of each line from the spatial chord distance to the ellipsoidal arc distance, and to the Transverse Mercator plane distance.



Table 1

## Reduction of Line Lengths

Line	Spatial Chord Dist. $D_C$ (meters)	Ellipsoidal Arc Dist. $d_A$ (meters)	TM Plane Dist. $d_M$ (meters)
Luke-ARPA	31397.907	31252.266	31251.588
Luke-Pier 2	3274.260	3272.955	3272.860
Luke-Puu Nianiau	28842.962	28769.051	28768.443
Luke-Kikalapuu	20699.239	20687.295	20686.788
ARPA-Puu Nianiau	7278.940	7214.236	7214.161
Kikalapuu-ARPA	11111.224	10871.767	10871.607
Pier 2-ARPA	30291.514	30132.223	30131.599
Pier 2-Puu Nianiau	27068.889	26984.017	26983.473
Pier 2-Kikalapuu	19349.021	19333.215	19332.759
Kikalapuu-Puu Nianiau	8745.333	8641.298	8641.175
Luke-Puu Olai	27459.365	27458.940	27458.162
Wailuku North-Luke	2469.155	2469.110	2469.037
Wailuku North-Pier 2	3482.199	3480.808	3480.707

Table 1. (continued) Reduction of Line Lengths

Line	Spatial Chord Dist. $D_C$ (meters)	Ellipsoidal Arc Dist. $d_A$ (meters)	TM Plane Dist. $d_M$ (meters)
ARPA-Mees	247.957	247.749	247.747
Mees-LURE	119.491	119.113	119.112
ARPA-LURE	227.299	226.668	226.666
LURE-Puu Nianiau	7349.130	7282.968	7282.893
Hana Waterpipe-Wolfe's Rock	629.559	629.145	629.157
Hana Waterpipe-Carter's Cliff	1986.969	1982.203	1982.244
Hana Waterpipe-Hana Airport	3474.112	3471.417	3471.490
Wolfe's Rock-Carter's Cliff	1989.646	1986.313	1986.356
Wolfe's Rock-Hana Airport	3023.884	3021.730	3021.795
Carter's Cliff-Hana Airport	2353.456	2353.450	2353.504

## LASER LINE ADJUSTMENTS

Since every type of measurement includes errors, measurement of the true value of an unknown quantity is not possible. Therefore, some method of obtaining the most probable value of the quantity in question from the measurements made is desired. It is also desirable that the same method should in some way minimize the differences between the observed quantities and the calculated most probable value. A method that meets the above criteria and is widely used is adjustment by least squares. In this type of adjustment the most probable value of the quantity is found by minimizing the sum of the squares of the differences between the observed quantities and the calculated most probable value.

In geodesy many types of measurements are made which must comply with certain geometrical conditions, e.g. the sum of the included angles of a triangle must equal  $180^{\circ}$ . However, since the measurements contain small errors, there will be discrepancies between the actual measurements and the geometrical conditions. In this case the process known as conditional adjustment by least squares is widely used. In conditional adjustment the geometrical conditions of the figures and the least squares principle must be simultaneously satisfied. Therefore, after applying the process the most probable value of each of the observations that also satisfy the geometrical conditions are obtained. In this way a discrepancy-free network is obtained from the adjustment. It must be emphasized that the adjusted

observations cannot be considered error-free, only that the errors in the measurements have been distributed in such a way that the discrepancies disappear. A discrepancy-free network is important in geodesy, since once it is obtained different types of coordinate computations can be made for each of the terminal points involved in the network.

Another important product obtained from conditional adjustment is the possibility for the investigation of the precision of the observations and results of the adjustment without the need for other observations with which to compare the adjusted results. The standard error of unit weight of a single observation can be found from the adjustments to each observation. The precision of the adjustment will be discussed in more detail later.

After the marker-to-marker spatial chord distances have been reduced to Hawaiian Transverse Mercator plane distances, each line contains small measuring errors, in addition to the negligible reduction errors. Certain types of closed figures can be formed from different combinations of the reduced laser lines to obtain the solutions. Also from the adjustment process the standard error of a single line measurement can be found.

On Maui two figures which are candidates for conditional adjustment can be formed from the laser lines which have been measured to date. As can be seen from Figure 1, there is an elongated quadrilateral with diagonals stretching from the Wailuku-Kahului area to the top of Mt. Haleakala. The terminal points of this figure are Luke, Pier 2, ARPA, and Puu Nianiau. The four lines terminating at Kikalapuu can also be combined with the frame of the quadrilateral

to give a quadrilateral with a center point. There is also a quadrilateral with diagonals located in the Hana Airport area, with terminal points of Hana Airport, Wolfe's Rock, Carter's Cliff, and Hana Waterpipe.

The above mentioned figures have errors associated with each of the line lengths. Because of these errors, the figures do not fit together perfectly. However, since every line of the figure is measured, the figures are considered to be over-determined. Because of this fact, conditions that the figures must meet can be developed. These conditions are developed from mathematical laws that must hold true. The number of conditions present,  $r$ , can be found from the formula:

$$r = n - u \quad (5)$$

where  $n$  is the total number of measurements made and,  $u$  is the number of measurements that make the figure completely determined.

The differences between the conditions derived from mathematical law and the actual measurements of the figure are called misclosures. This idea can be represented by the equations:

$$\begin{aligned} \sum a_j l_j - L_1 &= w_1 \\ \sum b_j l_j - L_2 &= w_2 \\ &\cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \\ \sum r_j l_j - L_r &= w_r, \quad j = 1, 2, 3, \dots, n \end{aligned} \quad (6)$$

where the  $(a_j, b_j, \dots, r_j)$  are known constants. In the equations  $l$  represents measured quantities,  $L$  represents the true quantities from mathematical law,  $w$  represents the misclosures,  $r$  is the number of conditions present, and  $n$  is the total number of measurements. The problem confronting us is to find the quantities  $v_1, v_2, \dots, v_n$  such that:

$$\begin{aligned} \sum a_j(1_j + v_j) - L_1 &= 0 \\ \sum b_j(1_j + v_j) - L_2 &= 0 \\ \cdot &\cdot \\ \cdot &\cdot \\ \cdot &\cdot \\ \sum r_j(1_j + v_j) - L_r &= 0 \end{aligned} \quad (7)$$

In other words we must try to find corrections to each measurement that when applied to the measurements will make the misclosures go to zero. However, since the number of corrections to be found is greater than the number of conditions present, the number of solutions possible is infinite. The basic premise we shall follow in picking a solution will be that the sum of the square of each correction will be a minimum:

$$\sum v_j^2 = \text{minimum} \quad (8)$$

better known as the least squares principle.

Letting  $V$  represent each measurement after it has been corrected, the condition equations can be written as:



Now notice that the first and third terms of each equation in Eq. 12 are the same as the left side of the equations of Eq. 6.

With this fact we obtain the following from Eq. 12:

$$\begin{aligned}
 \sum a_j v_j + w_1 &= 0 \\
 \sum b_j v_j + w_2 &= 0 \\
 \cdot &\cdot \\
 \cdot &\cdot \\
 \cdot &\cdot \\
 \sum r_j v_j + w_r &= 0
 \end{aligned}
 \tag{13}$$

The equations in Eq. 13 are called the reduced condition equations. They give the relationship between the corrections to be found and the misclosures of the original conditions in Eq. 7.

The final adjusted values of the observed quantities, after the obtained corrections have been applied, must meet the original conditions exactly. By the least squares principle these adjusted quantities will be the most probable values that will satisfy the conditions of the problem. Also, the sum of the squares of the corrections,  $\sum_{j=1}^n v_j^2$ , will be minimized. This fact must be satisfied simultaneously with the equations in Eq. 13.

Multiplying the reduced condition equations of Eq. 13 by  $-2 K_i$ ,  $i = 1, 2, 3, \dots, r$  respectively we get the following:



$$\begin{aligned}
- 2 K_1 \sum a_j v_j - 2 K_1 w_1 &= 0 \\
- 2 K_2 \sum b_j v_j - 2 K_2 w_2 &= 0 \\
\vdots & \\
- 2 K_r \sum r_j v_j - 2 K_r w_r &= 0
\end{aligned} \tag{14}$$

By adding Eq. 14 to Eq. 8 and collecting coefficients the following equation is obtained:

$$\begin{aligned}
\sum v_j^2 - 2K_1 \left[ \sum a_j v_j - w_1 \right] - 2K_2 \left[ \sum b_j v_j - w_2 \right] - \\
- 2K_r \left[ \sum r_j v_j - w_r \right] = \text{minimum}
\end{aligned} \tag{15}$$

For this equation to be minimized the partial derivatives with respect to each of the  $v$ 's must equal 0.

Therefore, taking the derivatives gives the following series of equations:

$$\begin{aligned}
2v_1 - 2 \left[ K_1 a_1 + K_2 b_1 + \dots + K_r r_1 \right] &= 0 \\
2v_2 - 2 \left[ K_1 a_2 + K_2 b_2 + \dots + K_r r_2 \right] &= 0 \\
\vdots & \\
2v_n - 2 \left[ K_1 a_n + K_2 b_n + \dots + K_r r_n \right] &= 0
\end{aligned} \tag{16}$$

From these the equations for each of the  $v$ 's can be obtained as:

$$\begin{aligned}
 v_1 &= K_1 a_1 + K_2 b_1 + \dots + K_r r_1 \\
 v_2 &= K_1 a_2 + K_2 b_2 + \dots + K_r r_2 \\
 &\vdots \\
 &\vdots \\
 &\vdots \\
 v_n &= K_1 a_n + K_2 b_n + \dots + K_r r_n
 \end{aligned}
 \tag{17}$$

Substituting the equations of Eq. 17 into Eq. 13 and combining coefficients, the following series of equations are obtained:

$$\begin{aligned}
 K_1 \sum a_j a_j + K_2 \sum a_j b_j + \dots + K_r \sum a_j r_j + w_1 &= 0 \\
 K_1 \sum a_j b_j + K_2 \sum b_j b_j + \dots + K_r \sum b_j r_j + w_2 &= 0 \\
 \vdots &\vdots \\
 K_1 \sum a_j r_j + K_2 \sum b_j r_j + \dots + K_r \sum r_j r_j + w_r &= 0
 \end{aligned}
 \tag{18}$$

These equations are known as the normal equations. If the misclosures in the normal equations are taken to the right side of the equal sign, the normal equations then form a  $r \times r$  square matrix in which the  $K$ 's are the unknowns.

The  $K$ 's are known in geodesy as correlates, and are found by solving the normal equations matrix by a suitable method. After the correlates are found, each correction,  $v_j$ , can then be found from Eq. 17. These corrections can then be applied to the corresponding observed quantities,  $l_j$ , to give the best values,  $V_j$ , of the quantities.

In the case of the conditional adjustment of the figures formed by the laser lines on Maui, three different types of figures for adjustment can be formed. The first, as stated before, is a quadrilateral with diagonals. The second is a quadrilateral with measured lines to an interior point. The third type is a quadrilateral with diagonals and an interior point combined.

In each case the number of conditions present comes from the number of overdeterminations of the figure. In the case of a quadrilateral with diagonals, any five of the measured lines fully determine the figure. However, since all six lines are measured, there is one overdetermination of the figure, and, therefore, there is one condition present. The case of the quadrilateral with an interior point also has one overdetermination and one condition present. In the case of the combined diagonals and interior point figure, however, there are three overdeterminations and, therefore, three conditions present.

In the case of a quadrilateral with all four sides and both diagonals measured, each of the four corner angles can be calculated as a function of the measured line lengths through the use of the law of cosines. The one condition is then the fact that the sum of the corner angles must equal  $360^\circ$ . (see Hirvonen, 1971, p. 91) Since each of the measured line lengths contain errors, the sum of the angles will introduce a misclosure,  $w_1$ :

$$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 - 360^\circ = w_1 \quad (19)$$

Figure 2 shows a sample quadrilateral with both diagonals and interior point and all angles labeled.

For the case of a quadrilateral with an interior point, each of the four angles around the interior point can be calculated as a function of the measured line lengths through use of the law of cosines. The one condition is then the fact that the sum of the angles around the interior point must equal  $360^\circ$ . As before, since

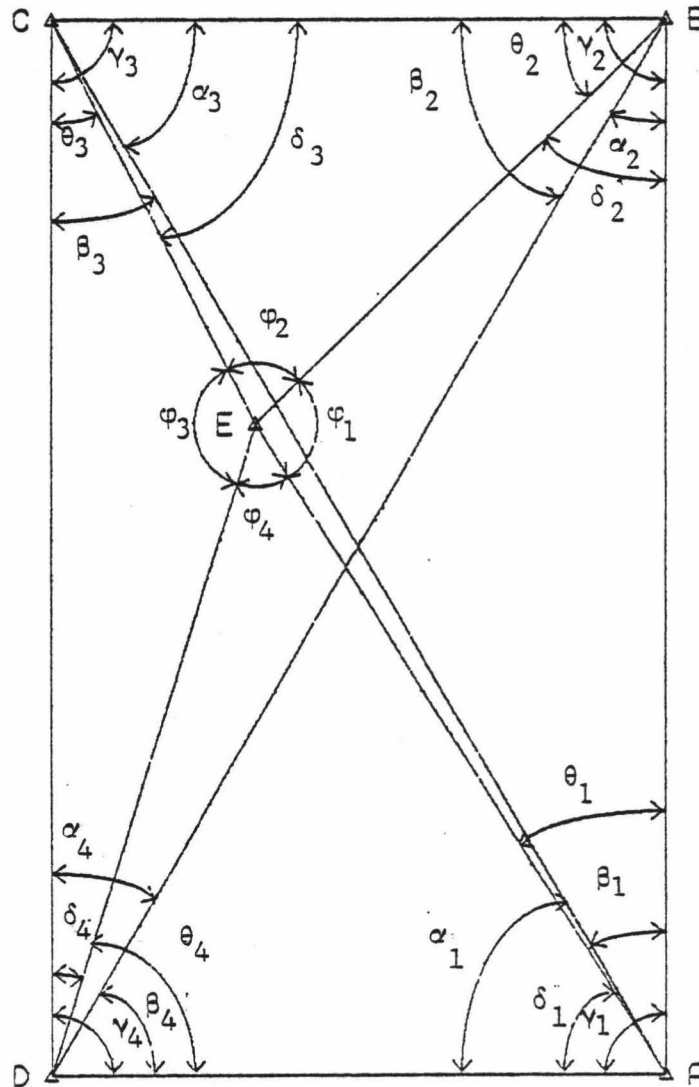


Figure 2. Sample Quadrilateral for Adjustments

the measured line lengths contain errors, the sum of the angles will introduce a misclosure for this case:

$$\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 - 360^\circ = w_1 \quad (20)$$

For the case involving the quadrilateral with both diagonals and interior point, three conditions must be found. The first condition is the same as the condition for the quadrilateral with diagonals, Eq. 19. The remaining two conditions come from the triangles with an interior point that can be formed, in each case, from two sides of the quadrilateral, one of the diagonals, and the interior point. From the sample quadrilateral in Figure 2, the two conditions would come from the triangles ACD and BCD along with the lines from each of the vertices to the interior point, E, in each case. The conditions, as can be expected, resemble the condition of the quadrilateral with an interior point, Eq. 20. The actual conditions from Figure 2 would be:

$$\begin{aligned} \text{AEC} + \varphi_3 + \varphi_4 - 360^\circ &= w_2 \\ \text{BED} + \varphi_2 + \varphi_3 - 360^\circ &= w_3 \end{aligned} \quad (21)$$

The next step in the conditional adjustment process is to find the coefficients,  $a_j$ ,  $b_j$ , ...,  $v_j$ , of Eq. 6. These coefficients are found by substituting into each of the condition equations the cosine law formula, involving line lengths, for each of the angles of the condition. In this manner the condition equations are made to be

functions of the measured line lengths. The partial derivatives of each of the conditions with respect to each of the lines involved in the condition is then taken. The coefficients are then calculated from each of the partial derivatives. In each of the cases discussed above with only one condition, there will be only one set of coefficients,  $a_1, a_2, \dots, a_n$ . However, in the case of, the quadrilateral combining diagonals and an interior point with three condition equations, there will be three sets of coefficients, a's, b's, and c s.

In geodesy the coefficients are usually designated by  $1/k_{XY}$ , where XY designates the line with respect to which of the partial differentials was taken. The k's represent normals from each of the lines to angles opposite them. (see Hirvonen, 1971, p. 91) The inverse length of these normals determine the value of the coefficients from which the normal equations are formed.

After the normal equations are formed from the coefficients, they are solved for the correlates. Since the number of conditions present and the number of correlates to be found are always equal, the cases of the quadrilateral with diagonals and with an interior point separately, will have only a single element matrix, and only one correlate to be found. The case of the quadrilateral with diagonals and an interior point combined, however, will have a 3 x 3 normal equation matrix, and three correlates to be found.

In each case after the correlates have been found, the corrections to each line can be easily found by using Eq. 17. The corrections are then added to each line, and the most probable length for each line is obtained.

As was mentioned before, the precision of the observations of the line length can be investigated from the results of the conditional adjustment. The standard error of a single observation of unit weight, usually designated by  $\sigma$ , can be found from the corrections by use of the following formula:

$$\sigma = \pm \left[ \sum v_j^2 / r \right]^{1/2} ; j = 1, 2, \dots, n \quad (22)$$

where  $n$  is the number of measured line lengths, and  $r$  is the number of conditions present in the adjustment.

Another way in which to obtain  $\sigma$  is from the product of the correlates and the condition equation misclosures:

$$\sum_{j=1}^n v_j^2 = - \sum_{i=1}^r K_i w_i \quad (23)$$

where  $n$  and  $r$  are as they were for Eq. 22. Substituting Eq. 23 into Eq. 22 gives:

$$\sigma = \pm \left[ - \sum K_i w_i / r \right]^{1/2} ; i = 1, 2, \dots, r \quad (24)$$

The use of Eq. 23 gives a valuable check to see that no errors have occurred during the computations done in the adjustment.

The Fortran program QUADAD2 in Appendix A does the total adjustment with only the measured length of the lines inputted. The explanation of the program tells the proper way the observed data must be inputted. After the program has adjusted the line lengths to the most probable values, it goes back and recomputes the angles used in the condition equations in order to check that the misclosures are now zero. The program also checks that Eq. 13 holds true, and that the angle conditions of the quadrilateral (to be discussed in the next section) have misclosures equal to zero. Also included in Appendix A is a sample output of the QUADAD2 program.

Another method of adjusting each of the different types of quadrilaterals was explored. In this method each of the angles labeled in Figure 2 are computed from the measured line lengths by use of the cosine law. The adjustment uses the least squares principle to find corrections to each of the angles in order to form a discrepancy-free network. As before since the measured line lengths contain errors, the angles computed from the line lengths will also contain errors, and any conditions formed will result in misclosures. After the conditions and misclosures are found, the adjustment closely resembles the line length adjustment. The  $a$ ,  $b$ , and  $c$  coefficients are found, the normal equations matrix is set up and solved, and the corrections to the angles are then calculated.

The angle conditions present in this type of adjustment are usually easily found through inspection of the figure keeping the geometrical rules in mind. There will be more conditions present in



this type of adjustment, however. Since a quadrilateral with diagonals can be formed from three independent triangles, there will be three angle conditions that can be found for the adjustment.

The three conditions used for the adjustment are:

$$\begin{aligned}\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 - 360^\circ &= w_1 \\ \alpha_3 + \beta_2 - \alpha_1 - \beta_4 &= w_2 \\ \alpha_4 + \beta_3 - \alpha_2 - \beta_1 &= w_3\end{aligned}\tag{25}$$

These three angle conditions are very common in this type of quadrilateral adjustment.

The second case of a quadrilateral with an interior point is slightly different. From Figure 2 it can easily be seen that the following angle conditions will be present:

$$\begin{aligned}\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 - 360^\circ &= w_1 \\ \theta_1 + \theta_2 + \theta_3 + \theta_4 + \delta_1 + \delta_2 + \delta_3 + \delta_4 - 360^\circ &= w_2\end{aligned}\tag{26}$$

The four individual triangles with the interior point as a common vertex will not appear as conditions, since when using the cosine law each of the three sides of each individual triangle are used in solving for each angle of that triangle, no misclosures are created in the four triangles.

In the final case of a quadrilateral with both diagonals and an interior point combined, the conditions will simply be a combination

of the two separate cases. This makes a total of five angle conditions present for the adjustment.

The next step in the adjustment of the angles is to find the coefficients,  $a$ ,  $b$ ,  $c$ , ..., for each of the conditions present. For angle conditions these coefficients are very easily found. The coefficient will be equal to either 1, -1, or 0, depending on the coefficient of the corresponding angle in each of the conditions of the adjustment.

As in the line length adjustment, after the proper coefficients have been found, the normal equations can be set up and solved for the correlates. From the correlates the correction to each angle can then be found and applied to arrive at an angular discrepancy-free network.

The precision of the measurement of each angle can be calculated by Eq. 22. A check on the calculations of the adjustment is also available from Eq. 24. In this case the dimensions of  $\sigma$  will be seconds of arc per angle measurement. The manner in which this standard error of an angle measurement can be compared with the standard error of a line length measurement will be discussed later.

As can be seen, the major drawback to this type of adjustment is the fact that the final products of the adjustment are angle measurements. However, the angles were not measured directly, but were calculated from measured line lengths, wherein our interest lies. Since there is no known baseline in our network, there is no way to calculate the adjusted line lengths from the adjusted angle measurements.

The Fortran Program QUADAD in Appendix A does all of the above calculations for line length input. The explanation of the program tells of the different types of input and options available to it. The program also checks to see that the condition equation residuals have gone to zero after adjustment. Also included in Appendix A is a sample output of the QUADAD program.

As was stated before the dimensions of  $\sigma$  obtained from the angle adjustment are seconds of arc per angle. In order to compare that standard error with the standard error obtained from the line length measurement, we must convert the dimensions from angular units to the dimensions of the line length standard error, i.e. millimeters per line.

From Figure 2 it can be seen that a change in any of the unadjusted angles will result in a change in the corresponding line opposing it. By finding the average length of the normals to each side of the line length adjustment, we will also have an idea of the average angle of the angle adjustment. The quantity  $\sigma$  can be thought of as the standard error of the average angle or line length. Therefore, by converting the  $\sigma$  with dimensions of seconds of arc per angle first to radians, then multiplying by the average length of a normal, we will have converted the dimensions to the desired millimeters per line.

Table 2 shows the standard error of a single line length measurement for each type of adjustment. It can be seen that in each case the  $\sigma$  from the line length adjustment is less than the  $\sigma$  from the angle adjustment. This would imply that the adjustment of line lengths gives a stronger network after adjustment.

Table 2  
Comparison of Standard Errors After Adjustment

Figure of Adjustment	$\sigma$ Angle Adj. (mm/line)	$\sigma$ Line Length Adj. (mm/line)
Quad. with diagonals	$\pm 7.79$	$\pm 4.35$
Quad. with interior point	$\pm 18.36$	$\pm 7.70$
Quad. with diagonals and interior point	$\pm 10.10$	$\pm 6.82$

The  $\sigma$  of the angle adjustments being greater in each case than the  $\sigma$  of the line length line adjustment can be thought of as that the angles of the angle adjustment need a greater amount of correcting than do the line lengths of the line length adjustment to reach a discrepancy-free network. The reason for this comes from the fact that the  $a, b, c, \dots$ , coefficients in the line length adjustment come from the length of the normal to each line. From the geometry of the quadrilateral a long line will have a short normal associated with it, while a short line will have a long normal. Since a long line will cumulate more error than a short line due to inherent measuring errors, a long line should have less weight associated with it, and will, therefore, need more correction than will a short line. In this way the normals act as a weighting function to each of the line lengths.

In the normal case of an angle adjustment process, each angle is actually measured, and the error involved in any angle is simply a function of each of the pointings. Therefore, when the angles

are measured directly, each angle will have the same weight. However, in our case the angles are solved for from the line lengths, and therefore, will not have equal weights. The weights of the angles will actually be a function of the length of the lines which make up each angle. The corrections are not equitably distributed, and the standard error of one measurement will not be minimized.

## COMPUTATION OF COORDINATES

In order to be able to detect any motion of any of the terminal points with respect to LURE, the adjusted line lengths were used to compute the coordinates of each terminal point that was tied to the adjusted quadrilateral. After each yearly measuring survey the new coordinates can then be computed and compared with the previous years coordinates to detect any relative motion between any of the terminal points. The procedure used in the coordinate computations was to first compute the Hawaiian Zone 2 Transverse Mercator plane coordinates from the adjusted laser lines. From the X and Y plane coordinates, the geographic coordinates,  $\phi$  and  $\lambda$ , on the Clarke 1866 reference ellipsoid are computed. Finally, the earth centered Universal Space Rectangular (USR) coordinates, X, Y, and Z, are computed by using the previously found geographic coordinates.

The computations proceeded in the following manner. The terminal point at Luke was chosen as the reference base of the entire system, while the line from Luke to Puu Nianiau was assigned as the azimuth reference for the system. The plane coordinates assigned to Luke were taken from USCGS Form 709, also approximate plane coordinates for Puu Nianiau were found in the same source. From the plane coordinates, the azimuth of Luke to Puu Nianiau is given by:

$$A_{L-PN} = \tan^{-1} \frac{\Delta X}{\Delta Y} \quad (27)$$

where  $\Delta X = X_{PN} - X_L$  and  $\Delta Y = Y_{PN} - Y_L$ . The plane coordinates of Puu Nianiau in the laser determined system is then given by:

$$X_{PN} = X_L + d_{L-PN} \sin A_{L-PN} \quad (28)$$

$$Y_{PN} = Y_L + d_{L-PN} \cos A_{L-PN}$$

where  $d_{L-PN}$  is the adjusted laser line length from Luke to Puu Nianiau on the HTM plane. Once this reference azimuth line has been established, the plane coordinates of any other point in the quadrilateral can be determined by the following: (see Laurila, 1976, pp. 210, 211)

$$X_A = X_L + X_A' \cdot \frac{\Delta X}{d_{L-PN}} - Y_A' \cdot \frac{\Delta Y}{d_{L-PN}} \quad (29)$$

$$Y_A = Y_L + X_A' \cdot \frac{\Delta Y}{d_{L-PN}} + Y_A' \cdot \frac{\Delta X}{d_{L-PN}}$$

where the subscript A designates the point for which the coordinates are being determined.

In Eq. 29  $X_A'$  and  $Y_A'$  are the plane coordinates of the point in question in the system with Luke as origin, and the line from Luke to Puu Nianiau as the positive X-axis.  $X_A'$  and  $Y_A'$  can be found from the following formulas:

$$X_A' = \frac{d_{L-A}^2 - d_{PN-A}^2 + d_{L-PN}^2}{2 d_{L-PN}} \quad (30)$$

$$Y_A' = \pm \left( d_{L-K}^2 - X_A'^2 \right)^{\frac{1}{2}}$$

The sign of  $Y_A'$  depends on which side of the auxiliary X-axis the point in question lies. The coordinates of any point not seen from both Luke and Puu Nianiau can be determined by setting up another baseline between two points whose plane coordinates have already been computed, and applying Eqs. 29 and 30.

The HTM Zone 2 map plane has latitude  $20^{\circ}20'$  assigned as the origin of the Y coordinates. The central meridian of  $156^{\circ}40'$  is assigned a scale factor of 0.999966667, and an X coordinate of 152,400.3 meters. Therefore, the formulas to compute the geographic coordinates,  $\phi$  and  $\lambda$ , from the HTM plane coordinates X and Y are modified in the following manner. (see Laurila, et al., 1969)

$$\begin{aligned} \phi &= \phi_1 - \frac{\tan \phi_1 \left[ \frac{\Delta X}{0.999966667} \right]^2}{2 M_1 N_1} \\ &+ \frac{\tan \phi_1 \left[ 5 + 3 \tan^2 \phi_1 \right] \left[ \frac{\Delta X}{0.999966667} \right]^4}{24 N_1^3 M_1} \quad (31) \\ \Delta \lambda &= \frac{\left[ \frac{\Delta X}{0.999966667} \right]}{N_1 \cos \phi_1} - \frac{\left[ 1 + 2 \tan^2 \phi_1 \right] \left[ \frac{\Delta X}{0.999966667} \right]^3}{6 N_1^3 \cos \phi_1} \end{aligned}$$

where  $\Delta X = X - 152,400.3$  and  $\lambda = 156^{\circ}40' - \Delta \lambda$ . Also in Eq. 31  $M_1$  is the radius of curvature along the meridian, and  $N_1$  is the radius of curvature in the prime vertical.  $M_1$  and  $N_1$  can be computed from the following:



$$M_1 = \frac{a(1-e^2)}{(1-e^2 \sin^2 \varphi_1)^{\frac{3}{2}}}$$

$$N_1 = \frac{a}{(1-e^2 \sin^2 \varphi_1)^{\frac{1}{2}}} \quad (32)$$

where  $a$  is the semimajor axis of the Clarke 1866 reference ellipsoid, and  $e^2$  is the eccentricity square of the same ellipsoid.

It is seen from Eqs. 31 and 32 that the quantity  $\phi_1$  is needed.  $\phi_1$  is known as the foot-point latitude, and is the parallel which crosses the central meridian at a distance  $Y$  from the equator. The foot-point latitude,  $\phi_1$ , is found by an iteration process from the following: (see Laurila, et. al., 1969)

$$\frac{Y/0.999966667 + 2249134.918}{a} =$$

$$(1 - \frac{1}{2}e^2 - 3/64e^4 - 5/256e^6) \varphi_1$$

$$- (3/8e^2 + 3/32e^4 + 45/1024e^6) \sin 2 \varphi_1 \quad (33)$$

$$+ (15/256e^4 + 45/1024e^6) \sin 4 \varphi_1 - 35/3072e^6 \sin 6 \varphi_1$$

where  $a$  and  $e^2$  are the same as before. In Eq. 33 the constant 2,249,134.918 is the number of meters from the equator to the 20°20' parallel.

Finally, from the geographic coordinates,  $\phi$  and  $\lambda$ , on the reference ellipsoid, the USSR coordinates,  $X$ ,  $Y$ , and  $Z$ , can be found from the following:

$$X = (N + H) \cos \phi \cos \lambda$$

$$Y = - (N + H) \cos \phi \sin \lambda \quad (34)$$

$$Z = (N(1 - e^2) + H) \sin \phi$$

where H is the elevation of the terminal point. In Eq. 34 N is again the curvature radius in the prime vertical given in Eq. 32 with  $\phi$  now replacing  $\phi_1$ .

The Fortran program GEOG in Appendix A does all of the above coordinate computations. Instructions as to input to the program are contained in the program itself. Tables 3 and 4 gives each of the three types of coordinates computed for each station in the system. (also in Berg et al., 1978)

Table 3

## HTM And Plane Coordinates of Each Laser Terminal

	HTM Plane Coordinates		Geographic Coordinates	
	X(meters)	Y(meters)	$\phi$	$\lambda$
Luke (base)	169,836.967	60,916.476	20°53'00".67847	156°29'56".70866
Puu Nianiau	196,029.256	49,017.108	20°46'32".18224	156°14'51".56398
ARPA	194,662.605	41,933.580	20°42'41".95232	156°15'39".42804
Pier 2	172,679.107	62,540.359	20°53'53".38016	156°28'18".30556
Kikalapuu	187,452.500	50,070.302	20°47'07".08275	156°19'48".01819
Lure	194,863.264	41,828.159	20°42'38".50781	156°15'32".50264
Mees	194,799.857	41,727.326	20°42'35".23409	156°15'34".70265
Wailuku North	169,287.755	63,323.975	20°54'18".98465	156°30'15".62664

Table 4

## USR Coordinates of Each Laser Terminal

Station	X(meters)	Y(meters)	Z(meters)
Luke (base)	-5,467,340.616	-2,377,371.116	2,259,250.272
Puu Nianiau	-5,462,451.366	-2,403,802.815	2,248,791.374
ARPA	-5,466,115.547	-2,403,901.352	2,242,502.749
Pier 2	-5,465,598.295	-2,379,714.623	2,260,732.143
Kikalapuu	-5,464,410.650	-2,395,296.294	2,249,322.253
Lure	-5,466,082.326	-2,404,105.766	2,242,409.067
Mees	-5,466,133.081	-2,404,058.510	2,242,311.759
Wailuku North	-5,466,776.796	-2,376,529.752	2,261,502.499

## SPATIAL INTERSECTION

After the line length adjustment was applied to the quadrilateral consisting of Luke, Pier 2, Puu Nianiau, and Kikalapuu, a  $\sigma$  of  $\pm 4$  millimeters was obtained from Eq. 22. Upon adding ARPA as a new corner point and using Kikalapuu as a center point, the  $\sigma$  obtained from the adjustment became  $\pm 38$  millimeters. This fact seemed to indicate that something was wrong with the assumed placement of the ARPA terminal point. Any error in the elevation is the largest source of error in the reduction from a spatial chord distance to a distance on the HTM plane. The elevation of ARPA, initially given as 3033.51 meters, was obtained by local differential leveling from HGS triangulation point Kolekole. The elevation of Kolekole, 3051.60 meters, is based on vertical angle trigonometric measurements made in 1876. For this reason the elevation of ARPA seemed suspect.

In order to find a better value for the elevation of ARPA, a spatial intersection procedure was applied to the terminal point. First the HTM plane coordinates were computed for the Luke, Pier 2, Kikalapuu, and Puu Nianiau terminal points from the adjusted line lengths of this quadrilateral according to Eqs. 27-30. From these plane coordinates the geographical coordinates,  $\phi$  and  $\lambda$ , were found from Eqs. 31-33. Finally the USR coordinates were found by using Eq. 34. These USR coordinates were then used to intersect the terminal point of ARPA from Luke, Kikalapuu, and Puu Nianiau terminals. The points at Pier 2, Kikalapuu, and Puu Nianiau were also used to intersect ARPA as a check.

The USR coordinates of ARPA were computed by using the spatial chord distances from Luke, Kikalapuu, and Puu Nianiaiu to ARPA to form a set of three equations and three unknowns. Using the Luke to ARPA line as an example the following can be obtained:

$$D_{L-A}^2 = (X_L - X_A)^2 + (Y_L - Y_A)^2 + (Z_L - Z_A)^2 \quad (35)$$

where  $X_A$ ,  $Y_A$ , and  $Z_A$  are the unknown USR coordinates of ARPA.

Equation 35 can be written as:

$$\begin{aligned} D_{L-A}^2 = & X_L^2 - X_L X_A - X_L X_A + X_A X_A + Y_L^2 - Y_L Y_A \\ & - Y_L Y_A + Y_A Y_A + Z_L^2 - Z_L Z_A + Z_L Z_A + Z_A Z_A \end{aligned} \quad (36)$$

and by assigning approximate values,  $\bar{X}_A$ ,  $\bar{Y}_A$ , and  $\bar{Z}_A$ , for  $X_A$ ,  $Y_A$ , and  $Z_A$  we can obtain the following equation:

$$\begin{aligned} D_{L-A}^2 = & X_L^2 - X_L \bar{X}_A - X_L \bar{X}_A + \bar{X}_A X_A + Y_L^2 - Y_L \bar{Y}_A \\ & - Y_L \bar{Y}_A + \bar{Y}_A Y_A + Z_L^2 - Z_L \bar{Z}_A - Z_L \bar{Z}_A + \bar{Z}_A Z_A \end{aligned} \quad (37)$$

From Eq. 37 we can finally obtain:

$$\begin{aligned} (X_L - \bar{X}_A) X_L + (Y_L - \bar{Y}_A) Y_L + (Z_L - \bar{Z}_A) Z_L - D_{L-A}^2 = \\ (X_L - \bar{X}_A) X_A + (Y_L - \bar{Y}_A) Y_A + (Z_L - \bar{Z}_A) Z_A \end{aligned} \quad (38)$$

where all terms on the left side of the equal sign are known constants. Two more equations of the form of Eq. 38 can be found by using the remaining two lines terminating at ARPA. By using an iterative process to solve the three equations of the form of Eq. 38, the USSR coordinates of ARPA can be closely approximated.

The longitude of ARPA can be found from the USSR coordinates by using the following formula:

$$\lambda_A = - \tan^{-1} (Y_A/X_A) \quad (39)$$

The latitude of ARPA is obtained from the following formula: (see Laurila, 1976, p. 197)

$$\varphi_A = \tan^{-1} \left[ \frac{a + \bar{H}}{b + \bar{H}} \right]^2 \cdot \frac{Z_A \sin \lambda_A}{X_A} \quad (40)$$

where  $\bar{H}$  is the approximate (original) elevation of ARPA, and a and b are the semimajor and semiminor axes of the Clark 1866 reference ellipsoid respectively.

Finally, the new elevation of ARPA is obtained from  $X_A$ ,  $Y_A$ , and  $Z_A$  through the following formulas:

$$\begin{aligned} H_A &= \frac{X_A}{\cos \varphi_A \cos \lambda_A} - N_A \\ H_A &= - \frac{Y_A}{\cos \varphi_A \sin \lambda_A} - N_A \\ H_A &= \frac{Z_A}{\sin \varphi_A} - N_A (1-e^2) \end{aligned} \quad (41)$$

where  $N_A$  is the radius in the prime vertical and is a function of  $\phi_A$ , and  $e^2$  is the eccentricity square of the reference ellipsoid. The Fortran program SPATIAL in Appendix A was written to do all of the above steps on a computer. Instructions on program usage and input precede the main body of the program.

After the completion of the above described spatial intersection, and a check using the Pier 2 terminal point instead of Luke, the mean value for the elevation of ARPA was found to be 3034.34 meters. This new elevation was then used in all computations until it can be checked by a first-order differential leveling survey. Upon applying the new elevation of ARPA to the conditional adjustment of a quadrilateral with an interior point the  $\sigma$  value after adjustment dropped from  $\pm 38$  millimeters to  $\pm 7$  millimeters.



### ERROR ANALYSIS OF LASER LINE COMPUTATIONS

Since the ultimate goal of the laser line measurements is to be able to detect any movement between any of the stations, it is very important that the errors introduced in each step of the process be identified. The magnitude of these errors and their cumulation as the process goes from spatial chord distances to HTM plane distances to HTM plane coordinates to geographic coordinates and, finally, to USR coordinates must be closely estimated.

As was stated before the instrument error with the Rangemaster II was found to be  $\pm 3$  mm, the error associated with frequency corrections, atmospheric corrections, etc. was estimated as  $2 \times 10^{-7}$  times the distance. These errors can be combined as shown in Eq. 42 to give a predicted measuring error for any line length.

$$\begin{aligned}
 m_i^2 &= \pm \left[ 3^2 + (0.2 \cdot d_i)^2 \right] \\
 &= \pm \left[ 9 + 0.04 \cdot d_i^2 \right]
 \end{aligned}
 \tag{42}$$

where  $m_i$  is given in millimeters and  $d_i$  is in kilometers. Table 5 shows each line, its unadjusted spatial chord length, and the predicted measuring error from Eq. 42.

The computed USR coordinates of each station were used to compute a check on the measured spatial chord distance. The average difference between the measured and computed spatial chord distances was found to be 0.9 mm, with 2.0 mm being the maximum difference seen. It can

be seen from Table 5 that the predicted measuring error for any line is much larger than the error introduced by the reduction process which can be considered negligible. It can be further concluded that after an annual remeasurement, for any change in the USSR coordinates of a station to be considered significant the change must be larger than the predicted measuring error for that line.

Table 5  
Lengths and Predicted Errors of Each Line

Line	Spatial Chord Length (km)	Predicted Error (mm)
Luke-ARPA	31.397907	± 7.0
Luke-Pier 2	3.274260	± 3.1
Luke-Puu Nianiau	28.842962	± 6.5
Luke-Kikalapuu	20.699239	± 5.1
ARPA-Puu Nianiau	7.278940	± 3.3
Kikalapuu-ARPA	11.111224	± 3.7
Pier 2-ARPA	30.291514	± 6.8
Pier 2-Puu Nianiau	27.068889	± 6.2
Pier 2-Kikalapuu	19.349021	± 4.9
Kikalapuu-Puu Nianiau	8.745333	± 3.5
Wailuku North-Luke	2.469155	± 3.0
Wailuku North-Pier 2	3.482199	± 3.1
ARPA-Mees	0.247957	± 3.0
Mees-Lure	0.119491	± 3.0
ARPA-Lure	0.227299	± 3.0
Lure-Puu Nianiau	7.349130	± 3.3

## GRAVITY MEASUREMENT NETWORKS

All gravity measurements were made with either of two gravity meters. The first meter to be used was a La Coste and Romberg Model G Geodetic Gravity Meter No. 1. Later measurements were made with a La Coste and Romberg Model G Geodetic Gravity Meter No. 144. Both of the gravity meters used are very similar in construction and operation. Both meters have a reading precision of  $\pm 0.01$  milligal. The meters have a weight on the end of a horizontal beam supported by a zero-length spring. The measuring system of the meters uses a measuring screw and lever system. Both meters have an internal heating unit to keep the temperature of the measuring system constant at  $52.2^{\circ}\text{C}$  for the G-1 and  $51.85^{\circ}\text{C}$  for the G-144.

Operating procedure is the same for both meters. After leveling the instrument, the measuring screw is turned to bring the crosshair viewed in the telescope eyepiece to the same spot on the scale for each reading. This means that for each measurement the weight is nulled at the same point. The reading taken is then a measure of the change of length of the spring between gravity points, and, therefore, of the change in gravity acting upon the weight. The correct calibration factors must be applied to the dial reading obtained at each point, before earth tide corrections and instrument drift correction are applied. After these corrections have been made differential gravity values in milligals can be found between each measured point.

The theoretical earth tide corrections were computed using a computer program assembled by H. C. Marsh of the Hawaii Institute of Geophysics in 1973. The program utilizes the Longman equations written for computers. The vertical component of the lunar tidal force is given as: (Woolard, et. al., 1973)

$$GM = \frac{\mu Mr}{d^3} (3 \cos^2 \theta - 1) + \frac{3}{2} \frac{\mu Mr}{d^4} (5 \cos^3 \theta - 3 \cos \theta) \quad (43)$$

where  $\mu$  is Newton's gravitational constant,  $M$  is the mass of the moon,  $r$  is the distance from the observation site to the center of the earth,  $d$  is the distance from the center of the earth to the center of the moon, and  $\theta$  is the zenith angle of the moon. The vertical component of the solar tidal force is given as:

$$GS = \frac{\mu Sr}{D^3} (\cos^2 \phi - 1) \quad (44)$$

where  $S$  is the mass of the sun,  $D$  is the distance from the center of the earth to the center of the sun,  $\phi$  is the zenith angle of the sun, and  $\mu$  and  $r$  are as above. The total vertical component is, therefore, given as:

$$GO = GM + GS \quad (45)$$

On Maui gravity measurements were taken at nine different points on the island to form a network. The location of the Maui gravity points can be seen in Figure 3, and further descriptions follow.

1. Kahului airport station is located in the terminal building along the wall near the airport manager's office. Station is marked by a small brass disk on the floor.

2. Pier 2 station is located at the Pier 2 laser terminal point which was described earlier, and next to the tidal gauge.

3. Gravity station #3 is located on the north side of Highway 37 approximately 0.3 miles southeast of milepost 3. The station is marked by a 12 inch square stepping stone set in concrete at ground level.

4. Gravity station #5 located at the junction of Highways 37 and 377. Station is in the middle of a triangular section of ground outlined by the junction of the two highways, and is marked by a 12 inch square concrete block set at ground level.

5. Haleakala National Park Headquarters station is located on a stone curb approximately 20 meters southwest of the entrance to the Park Headquarters building. Station is marked by an X chiseled into the stone curb.

6. Mees Solar Observatory station is located in the generator room of the solar observatory. Station is marked by a brass disk on the floor.

The six stations just described are all part of a calibration line established on Maui in the 1960's. The calibration line was

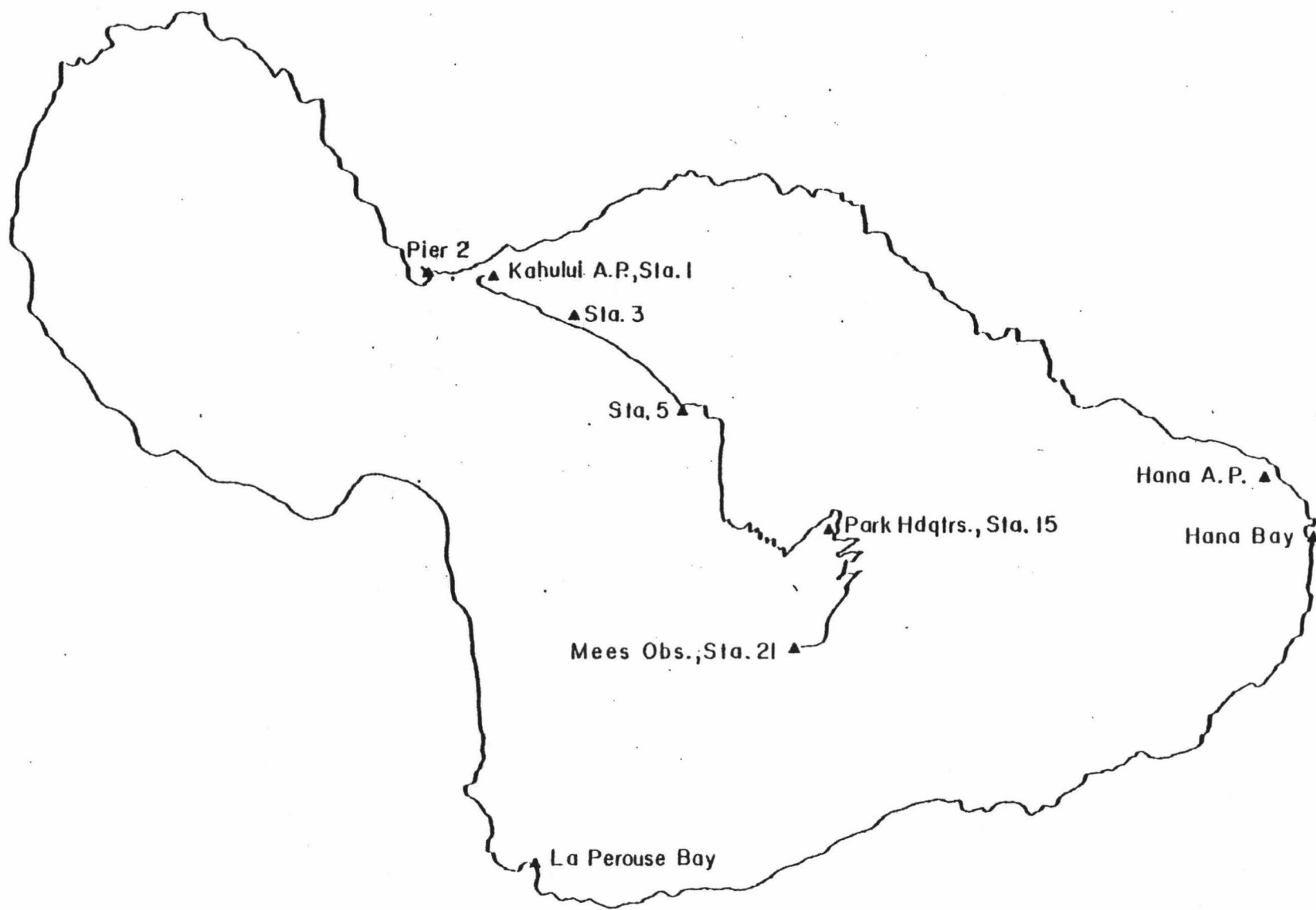


Figure 3. Location of Maui Gravity Stations

used to test the calibration constants of gravity meters. The Kahului airport station was the first of twenty-one points included in the line, while the Mees Solar Observatory station was the final point.

The descriptions of the final three Maui gravity network stations follow.

7. Hana Bay station is located on a concrete wall, which leads toward the pier at Hana Bay. The station is marked by a Phillip's head screw in a lead insert drilled into the concrete. The station coincides with the Hana Bay laser terminal, and is near the tidal gauge.

8. Hana Airport station is located at the southeast corner of an open lanai on the west side of the terminal building.

9. La Perouse Bay station is located at the end of Kihei Road by La Perouse Bay. The station is the southernmost of the three USCGS benchmarks located near the shoreline, and is near the tidal gauge.

On Oahu a nine station network for gravity measurements was set up. All of the points included in the network were in the Honolulu area. The points stretched from Hickam AFB east to St. Louis High School. Descriptions of each of the points follow.

1. The Hickam AFB station is located inside the Military Air Command terminal building on the floor next to the doors leading from the U. S. Customs inspection area.

2. The Inter-Island Terminal station is located at the Honolulu International Airport beside a stone pillar at the inter-island terminal building. The point is marked by a brass disk on the sidewalk.



3. Station #325 is located on the southernmost part of Sand Island on a concrete pier at the shoreline.

4. Station #324 is located south of the road leading to the state park on Sand Island. The point is on a concrete slab near a wire fence approximately 20 meters from the park entrance.

5. The Bishop Museum station is located in Room 2 of the Bishop Museum annex. The point is marked by a brass disk on the floor.

6. The Weights and Measures station is located in the Hawaii State Department of Agriculture Weights and Measures building at 1428 S. King Street. The point is on the floor beside a large generator in the east room of the building.

7. The Hawaii Institute of Geophysics station is located outside of HIG Room 108 near the wall. The point is marked by a brass disk on the sidewalk.

8. Station #47 is located on the north side of Lowrey Street directly across from a church. The point is on a 12 inch square concrete block next to the curb at ground level.

9. Station #171 is located on the west side of the road leading into St. Louis High School. The point is on a concrete block next to the curb at ground level approximately 50 meters from Waialae Avenue.

## GRAVITY NETWORK ADJUSTMENTS

The procedure utilized for each gravity network is the same. After measurements between the gravity stations have been made and corrected for earth tides and instrument drift, a value for the gravity difference between stations can be calculated. These gravity differences can then be used to form loop conditions for the network, which can then be put into a conditional adjustment process just as if a differential leveling loop were involved. After the adjustment process a discrepancy-free network is obtained along with the most probable values of the gravity differences between stations. If one of the stations of the network is assigned as a base along with an associated observed gravity value, the observed gravity value of each point can then be calculated.

If the above procedure is done yearly, any significant changes in absolute gravity values could be the signal for further investigation. Any changes in the absolute gravity value of a station could be caused by either a change in elevation or a change in the distribution of masses beneath the station. Since it is not desirable to have a costly first order differential leveling survey done yearly, the gravity network survey can be used as a rough leveling survey. If significant changes in the gravity values of the stations in the network are seen, it could be considered as a signal that a new differential leveling survey needs to be contracted. If after the differential leveling survey, no elevation changes of the stations

are measured, it would be assumed that the distribution of masses beneath the surface has changed, and other geophysical investigations are called for.

After all measured lines have been tabulated, the conditional equations for the adjustment can be found. Table 6 gives each measured line, the date the measurement was made, the gravity difference between the terminal points in milligals, and the instrument used for the Maui gravity network. Table 7 gives the same data for the Oahu gravity network. Figures 4 and 5 show the Maui and Oahu gravity networks, respectively, along with the number of measurements of each line.

When developing the conditional equations for the adjustment, I found it easiest to work with only triangular loops. After all measured lines have been used as one side of a loop, the number of conditions can be easily found by looking at the number of extra times each of the lines of the loop has been measured. It is most important to remember that only lines that have actually been measured can be used when setting up the conditional equations. Another important fact to be remembered is that each of the conditions must contain at least one measurement that does not occur in any of the other conditions. From the forty-three measured lines of the Maui network, there were thirty-five conditional equations that could be formed. The Oahu network contained forty measured lines and thirty-two conditions.

Table 6  
Maui Gravity Lines

Date	Line	Grav. Diff.	Inst.	#
8-76	1-3	+ 27.45	G-1	1
8-76	1-3	+ 27.44	G-1	2
9-76	1-3	+ 27.45	G-1	3
9-76	1-3	+ 27.46	G-1	4
5-77	1-3	+ 27.45	G-144	5
5-78	1-3	+ 27.44	G-1	6
5-78	1-3	+ 27.44	G-144	7
8-76	3-5	+ 68.52	G-1	8
8-76	3-5	+ 68.53	G-1	9
9-76	3-5	+ 68.53	G-1	10
9-76	3-5	+ 68.54	G-1	11
5-78	3-5	+ 68.52	G-1	12
5-78	3-5	+ 68.55	G-144	13
5-77	5-15	+ 321.92	G-144	14
5-78	5-15	+ 321.89	G-1	15
5-78	5-15	+ 321.89	G-144	16
5-77	15-21	+ 240.66	G-144	17
5-78	15-21	+ 240.65	G-1	18
5-78	15-21	+ 240.68	G-144	19
5-78	3-H.B.	- 78.92	G-144	20
8-76	H.B. -H.A.P.	+ 9.98	G-1	21
8-76	3-H.A.P.	- 69.01	G-1	22
5-78	3-H.A.P.	- 68.99	G-144	23
8-76	3-L.P.	- 37.45	G-1	24
8-76	3-L.P.	- 37.46	G-1	25

Table 6. (Continued) Maui Gravity Lines

Date	Line	Grav. Diff.	Inst.	#
9-76	1-L.P.	- 9.98	G-1	26
3-77	1-L.P.	- 9.97	G-1	27
5-78	5-L.P.	- 105.98	G-1	28
5-78	5-L.P.	- 105.98	G-144	29
5-78	15-L.P.	- 427.87	G-1	30
5-78	15-L.P.	- 427.91	G-144	31
5-78	2-21	+ 663.70	G-1	32
5-78	3-15	+ 390.41	G-1	33
5-78	2-3	+ 32.64	G-1	34
5-78	2-L.P.	- 4.84	G-144	35
5-78	2-L.P.	- 4.83	G-144	36
5-78	21-L.P.	- 668.58	G-144	37
5-78	2-5	+ 101.15	G-1	38
5-78	1-2	- 5.16	G-144	39
5-78	3-21	+ 631.08	G-144	40
5-78	3-21	+ 631.10	G-144	41
5-78	21-H.B.	- 710.00	G-144	42
5-78	21-H.A.P.	- 700.09	G-144	43

Table 7  
Oahu Gravity Lines

Date	Line	Grav. Diff.	Inst.	#
11-77	HIG-II	+ 25.80	G-1	1
11-77	HIG-II	+ 25.79	G-1	2
12-77	HIG-II	+ 25.83	G-1	3
12-77	HIG-BM	+ 5.87	G-1	4
11-77	HIG-W/M	+ 5.22	G-1	5
12-77	HIG-W/M	+ 5.25	G-1	6
12-77	HIG-#47	- 7.89	G-1	7
12-77	HIG-#171	+ 3.48	G-1	8
12-77	HIG-#324	+ 20.04	G-1	9
12-77	HIG-#325	+ 27.66	G-1	10
12-77	#47-#171	+ 11.36	G-1	11
5-78	#47-#171	+ 11.40	G-144	12
12-29	#47-W/M	+ 13.12	G-1	13
5-78	#47-W/M	+ 13.12	G-144	14
2-78	#47-BM	+ 13.74	G-1	15
12-77	#171-W/M	+ 1.73	G-1	16
12-77	#171-#325	+ 24.14	G-1	17
12-77	#171-#324	+ 16.54	G-1	18
12-77	#171-#324	+ 16.51	G-1	19
12-77	#325-BM	- 21.74	G-1	20
12-77	#325-II	- 1.85	G-1	21
12-77	#325-Hick	- 0.43	G-1	22
12-77	#325-#324	- 7.60	G-1	23
12-77	BM-W/M	- 0.63	G-1	24
12-77	BM-II	+ 19.92	G-1	25
11-77	II-W/M	- 20.57	G-1	26
12-77	II-Hick	+ 1.40	G-1	27
5-78	II-Hick	+ 1.37	G-144	28
5-78	II-Hick	+ 1.37	G-144	29
2-78	BM-Hick	+ 21.32	G-1	30
2-78	#171-Hick	+ 23.75	G-1	31
5-78	#171-BM	+ 2.38	G-144	32
5-78	BM-#324	+ 14.14	G-144	33
5-78	#324-II	+ 5.75	G-144	34
5-78	Hick-W/M	- 21.97	G-144	35
5-78	#171-II	+ 22.27	G-144	36
5-78	#324-Hick	+ 7.12	G-144	37
5-78	Hick-#47	- 35.06	G-144	38
5-78	#47-II	+ 33.69	G-144	39
5-78	Hick-HIG	- 27.21	G-144	40

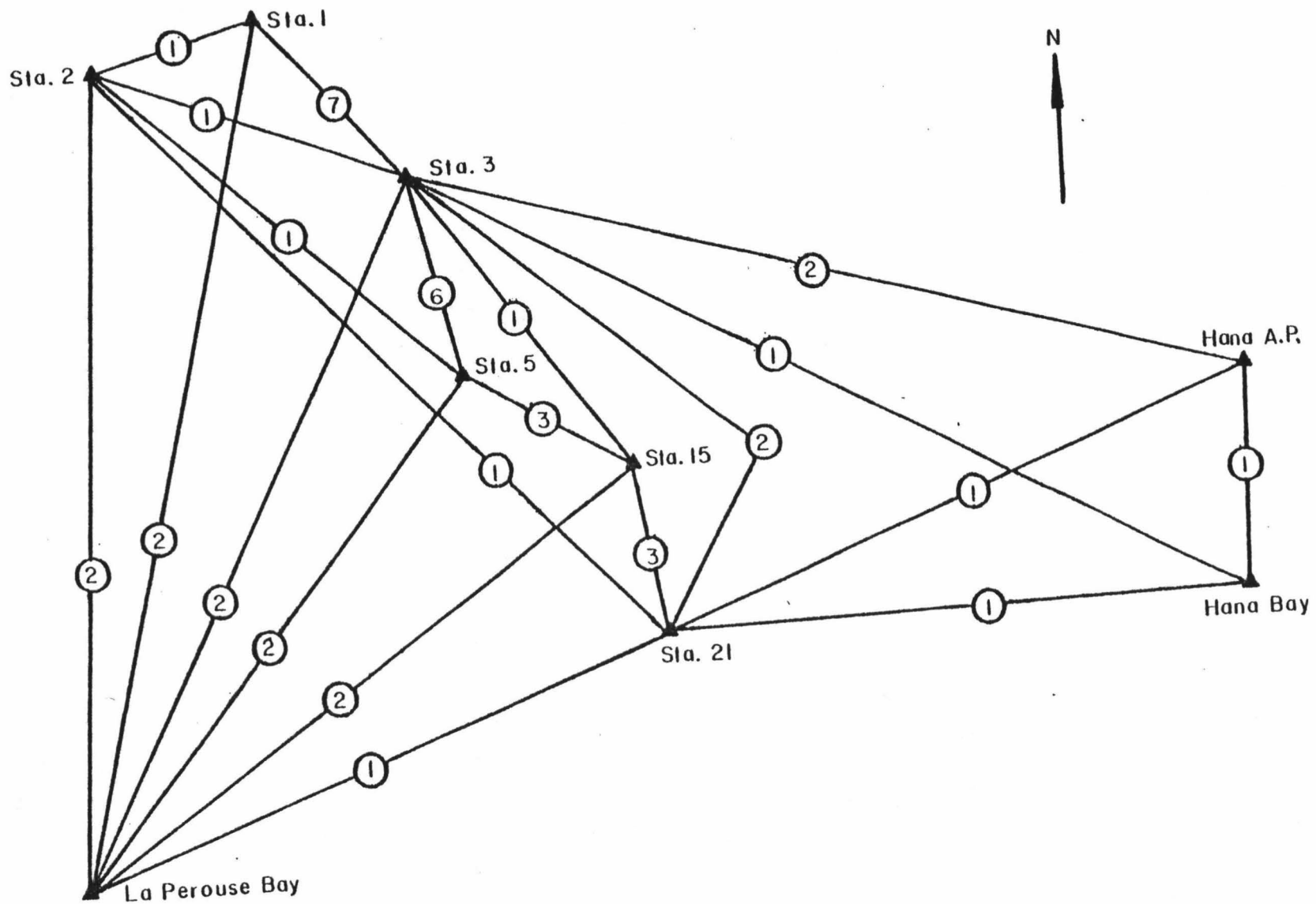


Figure 4. Maui Gravity Network .

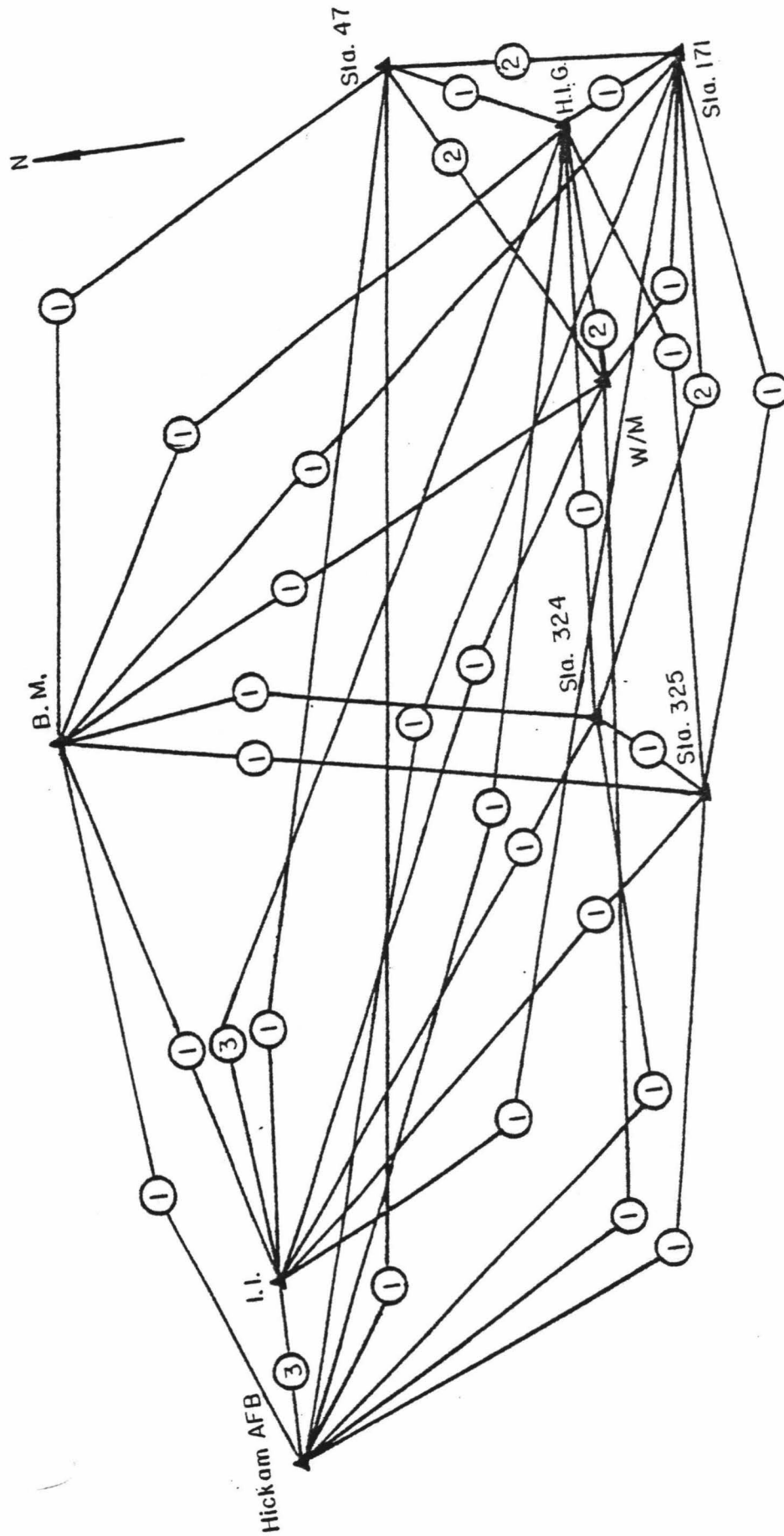


Figure 5. Oahu Gravity Network



After the conditional equations have been formed, the a, b, c coefficients must be found. These coefficients are easily found since they are equal to either 1, 0, or -1, depending upon the coefficient of the associated line in each of the conditional equations. After the above coefficients are found, the normal equations can be set up and solved to find the correlates. As before the correlates can then be used to find the correction to each of the measured gravity differences. The corrections can then be applied to the measured lines to find the most probable gravity difference for each of the measured lines. Once the most probable differences for each line have been found, the observed gravity value for each terminal point can be calculated from the observed gravity value assigned to the base point.

As in the quadrilateral adjustments, Eq. 22 or Eq. 24 can be used to calculate the standard error of one gravity difference measurement. In the cases of the Maui and Oahu gravity networks,  $\sigma$  can be thought of as a measure of the repeatability of a gravity measurement. Since each of the networks contained a large number of conditions,  $\sigma$  should give a good indication of the accuracy of the gravity meters, and of the strength of the entire network. Both networks gave a  $\sigma$  of  $\pm 0.02$  milligals.

The Fortran program GRAVNET in Appendix A was used to carry out the conditional adjustment described above. Instructions as to the input to the program are given in the program itself. The program solves the normal equations by the use of a Gaussian elimination

process. After the adjusted gravity differences are computed, the program goes back and checks to see that the original conditional equation residuals have gone to zero. Sample outputs of the program for the adjustment of the Maui and Oahu networks are in Appendix

The Inter-Island Terminal station at the Honolulu International Airport was chosen as base point for the Maui and Oahu gravity networks. The base was assigned an observed gravity value of 978933.03 milligals in the Potsdam system. Table 8 gives all terminal points of both networks, the adjusted gravity difference from the base point, and the absolute gravity value of each station.

All gravity values utilized in this study are referenced to the International Gravity Base Network (Woollard and Rose, 1963). It should be remarked that Woollard (1978) has presented the change in theoretical gravity values brought about by the recent adoption of the new reference ellipsoid (Geodetic Reference System, 1967) here referred to as GRS 67. In addition, he has evaluated the reliability of the gravity standard and Potsdam datum correction incorporated in the values of the International Gravity Standardization Net (IGSN 71) prepared by Morelli et al (1974). The observed gravity based on Woollard and Rose (1963) values can be revised to be compatible with the IGSN 71 by subtracting 14.7 mgal.

Table 8  
Differences from Base and Observed Gravity Values

Gravity Pt.	Diff. from Base (milligals)	Obs. Gravity (milligals)
#1, KAP	- 43.52	978889.51
#2, Pier 2	- 38.35	978894.68
#3	- 70.97	978862.06
#5	- 139.50	978793.53
#15, Park Hdqts.	- 461.40	978471.63
#21, Mees Obs.	- 702.06	978230.97
La Perouse Bay	- 33.52	978899.51
Hana A.P.	- 1.98	978931.05
Hana Bay	+ 7.96	978940.99
Inter Isle (base)	0	978933.03
Hickam AFB	- 1.39	978931.64
#324	+ 5.77	978938.80
#325	- 1.84	978931.19
Bishop Museum	+ 19.92	978952.95
W/M	+ 20.57	978953.60
HIG	+ 25.80	978958.83
#47	+ 33.68	978966.71
#171	+ 22.30	978955.33

## ERROR ANALYSIS OF GRAVITY MEASUREMENTS

As far as errors in the gravity networks go, the  $\sigma$  value that comes out of the adjustment of each network gives a strong indication of the error included in one gravity difference measurement. After annual measurements of the gravity networks have been made and adjusted, and new observed gravity values have been calculated for each station, any deviations from past years values can be investigated. A change could not be considered significant unless the difference between the values is greater than the  $\sigma$  value of the network. In the case of the adjustments included here with  $\sigma$  values of  $\pm 0.02$  milligals, a difference of 0.02 milligals or less could not be considered to be changed. A difference of 0.02 - 0.03 milligals might be considered a questionable change, and may need closer observation in the future. A difference greater than 0.03 milligals could be considered significant, and could warrant further investigation as to its cause.

In Table 9 the 1965 gravity values for five points of the Maui calibration line are given along with the standard deviation of each of the measurements of that station computed for the mean value of six different measurements. The 1977-1978 values are also shown along with the standard deviation of one measurement from the network adjustment. The error limits given with the difference of the gravity values are found by adding the standard deviations of the two values quadratically. As can be seen, only two of the points can be considered to have shown significant changes. The Park Headquarters

Table 9

1965 and Present Gravity Values of Five Stations  
on the Maui Calibration Line

Station	1965 Value (milligals)	1977-1978 Value (milligals)	Diff. (milligals)
Inter-Island (base)	978,933.03	978,933.03	0
#1; Kahului A.P.	978,339.51 ± 0.02	978,889.51 ± 0.017	0.00 ± 0.03
#3	978,862.11 ± 0.01	978,862.06 ± 0.017	+ 0.05 ± 0.02
#5	978,793.52 ± 0.02	978,793.53 ± 0.017	- 0.01 ± 0.03
#15, Park Hdqts.	978,471.50 ± 0.03	978,471.63 ± 0.017	- 0.13 ± 0.03
#21, Mees Obs.	978,230.94 ± 0.05	978,230.97 ± 0.017	- 0.03 ± 0.05

station definitely deserves closer observation and investigation. The change at station #3 can be considered questionable since the station had been destroyed, and the relocation was done without perfect elevation control.

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APPENDIX A

ALPHABETICAL LISTING OF  
COMPUTER OUTPUTS

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GEOG . . . . .	91
GRAVNET . . . . .	98
MAUI GRAVITY NETWORK . . . . .	104
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REDUC . . . . .	68
SAMPLE OUTPUT OF QUADAD . . . . .	89
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SPATIAL . . . . .	95

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C          **** REDUC ****
C
C          PROGRAM TO REDUCE A SPATIAL DISTANCE TO A UNIVERSAL TRANS-
C          VERSE MERCATOR PLANE DISTANCE. WRITTEN BY B. SCHENCK; AUG. 1977.
C
C          THE PROGRAM TAKES TWO DATA CARDS AS INPUT FOR EACH SEPARATE
C          LINE TO BE COMPUTED. THE FIRST CARD OF THE PAIR MAY CONTAIN THE
C          NAME OF THE LINE, AND WILL BE PRINTED AT THE TOP OF THAT SECTION
C          OF THE OUTPUT. THE SECOND CARD OF EACH PAIR MUST CONTAIN THE
C          FOLLOWING: COLUMNS 1-19 CONTAIN THE SPATIAL CHORD DISTANCE,
C          "BIGDC", IN AN F10.3 FORMAT; COLS. 11-30 CONTAIN THE ELEVATIONS
C          OF THE TWO ENDPOINTS, "H1" & "H2", IN TWO F10.2 FORMATS; COLS.
C          31-50 CONTAIN THE "X" PLANE COORDINATES OF EACH POINT OF THE
C          LINE, IN TWO F10.1 FORMATS; COLS. 51-55 CONTAIN THE AZIMUTH OF
C          THE LINE, IN AN F5.1 FORMAT; COLS. 56-75 CONTAIN THE LATITUDE IN
C          DECIMAL DEGREES OF EACH POINT, IN TWO F10.6 FORMATS. THE FINAL
C          DATA CARD MUST CONTAIN A "1" IN COLUMN #80. OUTPUT FOR EACH LINE
C          IS: THE INFORMATION ON THE FIRST DATA CARD OF EACH PAIR; THE
C          SPATIAL CHORD DISTANCE; THE ELEVATIONS OF THE ENDPOINTS; THE
C          ELLIPSOIDAL ARC DISTANCE, "DA"; AND THE UTM MAP DISTANCE, "DM2".
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C### THIS SECTION INITIALIZES VARIABLES AND READS INPUT.
      DIMENSION XLINE(12)
      DOUBLE PRECISION BIGDC,H1,H2,RA,W,DSIN,ESQ,AZ,X1,X2,DSQRT,
      IGLAT1,GLAT2,GLAT,DCOS,PI,REDUCT,U,DAA,DA,DM2,SGLAT,AM,AN,A
      REDUCT=0.9999667D 0
      PI=3.1415926535898D 0
      A=6378206.4D 0
      ESQ=0.006768658D 0
      200 READ (8,103) (XLINE(I),I=1,12)
      103 FORMAT (12A6)
      WRITE (9,104)
      104 FORMAT (' LINE REDUCTION ONTO TM PLANE OF:')
      WRITE (9,103) XLINE
      READ (8,100) BIGDC,H1,H2,X1,X2,AZ,GLAT1,GLAT2,N
      100 FORMAT (F10.3,2F10.2,2F10.1,F5.1,2F10.6,T80,I1)
      WRITE (9,101) BIGDC,H1,H2
      101 FORMAT (' SPATIAL CHORD DISTANCE =',5X,F10.3,/, ' ELEVATIONS OF END
      1POINTS ARE:',5X,F8.2,2X, 'AND',5X,F8.2)
C### THIS SECTION COMPUTES THE MEAN OF THE LATITUDE OF THE ENDPOINTS,
C### CONVERTS IT TO RADIANS AND TAKES THE SINE OF THE RESULT.
      GLAT=(GLAT1+GLAT2)/2./180.*PI
      SGLAT=DSIN(GLAT)**2
C### THIS SECTION COMPUTES "W" USING THE FORMULA:
C###  $W = \sqrt{1 - ESQ * (\sin(GLAT))^2}$ 
      W=DSQRT(1.-ESQ*(SINE(GLAT)**2))
      W=DSQRT(1.-ESQ*SGLAT)
C### THIS SECTION COMPUTES "M" & "N" USING THE FORMULAS:
C###  $M = A(1 - E^{**2}) / W^{**3}$ ;  $N = A / W$ , WHERE "A" IS THE SEMIMAJOR AXIS OF
C### THE ELLIPSOID.
      AM=A*(1.+(-ESQ))/W**3
      AN=A/W
C### THIS SECTION CONVERTS THE AZIMUTH OF THE LINE TO RADIANS, THEN
C### COMPUTES THE CURVATURE RADIUS USING THE FORMULA:
C###  $RA = M*N / (M*SINE(AZ)**2 + N*COSINE(AZ)**2)$ 
      AZ=AZ/180.*PI
      RA=AM*AN/(AM*DSIN(AZ)**2+AN*DCOS(AZ)**2)
      WRITE (9,105) RA
      105 FORMAT (' CURVATURE RADIUS =',F12.2)
C### THIS SECTION COMPUTES "U", THE ELLIPSOIDAL ARC DISTANCE, AND THE
C### UTM MAP DISTANCE USING THE FORMULAS:

```

```
C***      U=DC**2-(H1-H2)**2;
C***      DA=SQRT(12.*RA**2*U/(12.*(RA+H1)*(RA+H2)-U));
C***      UTM=DA*(0.9999667+((X1**2+X1*X2+X2**2)/6./RA**2)).
          U=BIGDC**2-(H1-H2)**2
          DAA=12.*RA**2*U/(12.*(RA+H1)*(RA+H2)-U)
          DA=DSQRT(DAA)
          DM2=DA*(REDUCT+((X1**2+X1*X2+X2**2)/6./RA**2))
          WRITE (9,107) DA,DM2
107  FORMAT (' ELLIPSOIDAL ARC DISTANCE (EQ. 15.25) =',2X,F10.3,/, ' UTM
1  MAP DISTANCE =',2X,F10.3,/)
          IF (N.NE. 1) GO TO 200
          STOP
          END
```



```

    DIMENSION XADJ(20),MAM(6,4),MAD(6,4),K(3),AR(6,4),AQ(6,4),AM(6,4),
    IAS(6,4),W(3),P(10),R(10,3),CC(3,3),S(10),WW(3),CLOSE(3),Z(5),WS(3)
    EQUIVALENCE (AR(6,2),BEC),(AR(6,3),CED),(AR(6,4),DEA),(AR(1,1),A),
    1(AR(1,2),B),(AR(1,3),C),(AR(1,4),D),(AR(6,1),AEB),(NN,N),(P(1),PAB
    2),(P(2),PBC),(P(3),PCD),(P(4),PAD),(P(5),PAC),(P(6),PBD),(P(7),PAE
    3),(P(8),PBE),(P(9),PCE),(P(10),PDE)
    DATA R,CC/30*0.D0,9*0.D0/
    READ (8,100) XADJ
100  FORMAT (20A4)
    READ (8,101) NTYPE
101  FORMAT (T7,I1)
    PI=3.1415926535898D00
    NO=0
C*** NSID = NUMBER OF SIDES OF FIGURE BEING ADJUSTED
C*** NAN = NUMBER OF DIFFERENT TYPES OF ANGLES IN FIGURE BEING ADJUSTED;
C*** I.E. ALPHAS, BETAS, GAMMAS, ETC.
    NSID=4
    NNN=1
    NAN=3
    IF (NTYPE .EQ. 2) NNN=4
    IF (NTYPE .NE. 1) NAN=6
    N=1
    IF (NTYPE .EQ. 3) N=3
    NL=10
    IF (NTYPE .EQ. 1) NL=6
    WRITE (9,100) XADJ
    READ (8,102) AB,AC,AD,AE,BC
    READ (8,102) BD,BE,CD,CE,DE
102  FORMAT (5F10.7)
    WRITE (9,103) AB,AC,AD,AE,BC,BD,BE,CD,CE,DE
103  FORMAT (/, ' MEASURED LINE LENGTHS: ',/, ' AB=',F10.6,/, ' AC=',F10.6,
    1/, ' AD=',F10.6,/, ' AE=',F10.6,/, ' BC=',F10.6,/, ' BD=',F10.6,/, ' BE
    2=',F10.6,/, ' CD=',F10.6,/, ' CE=',F10.6,/, ' DE=',F10.6)
104  CONTINUE
C*** THIS SECTION COMPUTES EACH ANGLE OF THE FIGURE IN RADIANS AND IN
C*** DEGREES, MINUTES, AND SECONDS, BY USE OF THE COSINE LAW.
    IF (NTYPE .EQ. 2) GO TO 105
    AR(1,1)=ANGLE(AD,AB,BD)
    AR(1,2)=ANGLE(AB,BC,AC)
    AR(1,3)=ANGLE(BC,CD,BD)
    AR(1,4)=ANGLE(CD,AD,AC)
    AR(2,1)=ANGLE(AC,AD,CD)
    AR(2,2)=ANGLE(AB,BD,AD)
    AR(2,3)=ANGLE(AC,BC,AB)
    AR(2,4)=ANGLE(BD,CD,BC)
    AR(3,1)=ANGLE(AB,AC,BC)
    AR(3,2)=ANGLE(BC,BD,CD)
    AR(3,3)=ANGLE(AC,CD,AD)
    AR(3,4)=ANGLE(AD,BD,AB)
105  CONTINUE
    IF (NTYPE .EQ. 1) GO TO 106
    AR(4,1)=ANGLE(AD,AE,DE)
    AR(4,2)=ANGLE(AB,BE,AE)
    AR(4,3)=ANGLE(BC,CE,BE)
    AR(4,4)=ANGLE(CD,DE,CE)
    AR(5,1)=ANGLE(AB,AE,BE)
    AR(5,2)=ANGLE(BE,BC,CE)
    AR(5,3)=ANGLE(CE,CD,DE)
    AR(5,4)=ANGLE(AD,DE,AE)
    AR(6,1)=ANGLE(AE,BE,AB)
    AR(6,2)=ANGLE(BE,CE,BC)
    AR(6,3)=ANGLE(CE,DE,CD)
    AR(6,4)=ANGLE(AE,DE,AD)
    AEC=ANGLE(AE,CE,AC)

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BED=ANGLE(BE,DE,BD)
ACE=ANGLE(AC,CE,AE)
CAE=ANGLE(AC,AE,CE)
DBE=ANGLE(BD,BE,DE)
BDE=ANGLE(BD,DE,BE)
106 CONTINUE
DO 107 I=NNN,NAN
DO 107 J=1,NSID
AQ(I,J)=AR(I,J)*180./PI
MAD(I,J)=AQ(I,J)
AM(I,J)=(AQ(I,J)-MAD(I,J))*60.
MAM(I,J)=AM(I,J)
107 AS(I,J)=(AM(I,J)-MAM(I,J))*60.
IF (NO .EQ. 0) WRITE (9,111)
111 FORMAT (/,' CALCULATED ANGLES (RADIANS):',T40,'CALCULATED ANGLES (
1DEGREES):')
IF (NO .EQ. 1) WRITE (9,142)
142 FORMAT (/,' ADJUSTED ANGLES (RADIANS):',T40,'ADJUSTED ANGLES (DEGR
1EES):')
IF (NTYPE .EQ. 1) WRITE (9,108) ((AR(I,J),MAD(I,J),MAM(I,J),AS(I,J
1),J=1,NSID),I=NNN,NAN)
IF (NTYPE .EQ. 2) WRITE (9,109) ((AR(I,J),MAD(I,J),MAM(I,J),AS(I,J
1),J=1,NSID),I=NNN,NAN)
IF (NTYPE .EQ. 3) WRITE (9,110) ((AR(I,J),MAD(I,J),MAM(I,J),AS(I,J
1),J=1,NSID),I=NNN,NAN)
108 FORMAT (/,' GAMMA 1-4:',T40,'GAMMA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X
1,F7.3,/),' ALPHA 1-4:',T40,'ALPHA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,
2F7.3,/),' BETA 1-4:',T40,'BETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.
33,/))
109 FORMAT (/,' DELTA 1-4:',T40,'DELTA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X
1,F7.3,/),' THETA 1-4:',T40,'THETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,
2F7.3,/),' PHI 1-4:',T40,'PHI 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3,
3/))
110 FORMAT (/,' GAMMA 1-4:',T40,'GAMMA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X
1,F7.3,/),' ALPHA 1-4:',T40,'ALPHA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,
2F7.3,/),' BETA 1-4:',T40,'BETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.
33,/),' DELTA 1-4:',T40,'DELTA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3
4,/),' THETA 1-4:',T40,'THETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3,
5/),' PHI 1-4:',T40,'PHI 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3,/))
C*** THIS SECTION CONTAINS THE CONDITIONS AND COMPUTES THE RESIDUALS
C*** BEFORE ADJUSTMENT IN RADIANS AND SECONDS.
IF (NTYPE .NE. 2) W(1)=A+B+C+D-2.*PI
IF (NTYPE .EQ. 2) W(1)=AEB+BEC+CED+DEA-2.*PI
IF (NTYPE .NE. 3) GO TO 112
W(2)=AEC+BEC+AEB-2.*PI
W(3)=BED+BEC+CED-2.*PI
112 CONTINUE
DO 150 I=1,N
150 WS(I)=W(I)*180./PI*3600.
IF (NO .EQ. 0) WRITE (9,113)
113 FORMAT (/,' ANGLE CONDITION CLOSURES (RADIANS):')
IF (NO .EQ. 1) WRITE (9,143)
143 FORMAT (/,' ANGLE CONDITION CLOSURES AFTER ADJUSTMENT (RAD):')
WRITE (9,114) (I,W(I),WS(I),I=1,NN)
114 FORMAT (I3,' :',D12.4,' = ',F7.3,' "')
IF (NO .EQ. 1) GO TO 144
DO 125 I=1,N
125 WW(I)=W(I)
C*** THIS SECTION COMPUTES THE NORMALS TO EACH LINE IN KILOMETERS.
IF (NTYPE .EQ. 2) GO TO 120
KAB=KORM1(AB,AD,BC,A,B)
KCD=KORM1(CD,AD,BC,D,C)
KBC=KORM1(BC,AB,CD,B,C)
KAD=KORM1(AD,AB,CD,A,D)

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KAC=-KORM2(AC,AB,BC,B,CD,AD,D)
KBD=-KORM2(BD,AD,AB,A,BC,CD,C)
IF (NTYPE .NE. 2) GO TO 115
120 CONTINUE
KAB=-KORM3(AB,AE,BE,AEB)
KBC=-KORM3(BC,BE,CE,BEC)
KCD=-KORM3(CD,CE,DE,CED)
KAD=-KORM3(AD,DE,AE,DEA)
KAE=KORM1(AE,DE,BE,AR(6,4),AR(6,1))
KBE=KORM1(BE,AE,CE,AR(6,1),AR(6,2))
KCE=KORM1(CE,BE,DE,AR(6,2),AR(6,3))
KDE=KORM1(DE,CE,AE,AR(6,3),AR(6,4))
115 CONTINUE
IF (NTYPE .NE. 3) GO TO 121
KAB2=-KORM3(AB,AE,BE,AEB)
KBC2=-KORM3(BC,BE,CE,BEC)
KAC2=-KORM3(AC,AE,CE,AEC)
KAE2=KORM1(AE,CE,BE,AEC,AEB)
KBE2=KORM1(BE,AE,CE,AEB,BEC)
KCE2=KORM1(CE,BE,AE,BEC,AEC)
KBD3=-KORM3(BD,BE,DE,BED)
KBC3=-KORM3(BC,BE,CE,BEC)
KCD3=-KORM3(CD,CE,DE,CED)
KDE3=KORM1(DE,CE,BE,CED,BED)
KBE3=KORM1(BE,DE,CE,BED,BEC)
KCE3=KORM1(CE,BE,DE,BEC,CED)
121 CONTINUE
WRITE (9,116)
116 FORMAT (/, ' NORMALS TO LINES (KMD:)' )
WRITE (9,117) KAB,KBC,KCD,KAD
117 FORMAT (/, ' KAB=',F10.6,/, ' KBC=',F10.6,/, ' KCD=',F10.6,/, ' KAD=',
1F10.6)
IF (NTYPE .NE. 2) WRITE (9,118) KAC,KBD
118 FORMAT ( ' KAC=',F10.6,/, ' KBD=',F10.6)
IF (NTYPE .EQ. 2) WRITE (9,119) KAE,KBE,KCE,KDE
119 FORMAT ( ' KAE=',F10.6,/, ' KBE=',F10.6,/, ' KCE=',F10.6,/, ' KDE=',
1F10.6)
IF (NTYPE .EQ. 3) WRITE (9,122) KAB2,KBC2,KAC2,KAE2,KBE2,KCE2,KBC3
1, KBD3, KCD3, KBE3, KCE3, KDE3
122 FORMAT ( ' KAB2=',F10.6,/, ' KBC2=',F10.6,/, ' KAC2=',F10.6,/, ' KAE2=
1',F10.6,/, ' KBE2=',F10.6,/, ' KCE2=',F10.6,/, ' KBC3=',F10.6,/, ' KBD
23=',F10.6,/, ' KCD3=',F10.6,/, ' KBE3=',F10.6,/, ' KCE3=',F10.6,/, ' K
3DE3=',F10.6)
C*** THIS SECTION COMPUTES THE WEIGHT OF EACH LINE ACCORDING TO THE
C*** DESIRED WEIGHT FUNCTION.
PAB=WEIGHT(AB)
PBC=WEIGHT(BC)
PCD=WEIGHT(CD)
PAD=WEIGHT(AD)
PAC=WEIGHT(AC)
PBD=WEIGHT(BD)
PAE=WEIGHT(AE)
PBE=WEIGHT(BE)
PCE=WEIGHT(CE)
PDE=WEIGHT(DE)
WRITE (9,123) PAB,PBC,PCD,PAD,PAC,PBD,PAE,PBE,PCE,PDE
123 FORMAT (/, ' INVERSE WEIGHTS OF LINES:',/, ' PAB=',F8.3,/, ' PBC=',F8
1.3,/, ' PCD=',F8.3,/, ' PAD=',F8.3,/, ' PAC=',F8.3,/, ' PBD=',F8.3,/, '
2 PAE=',F8.3,/, ' PBE=',F8.3,/, ' PCE=',F8.3,/, ' PDE=',F8.3)
C*** THIS SECTION INITIALIZES THE CORRELATE MATRIX.
IF (NTYPE .EQ. 2) GO TO 131
C***CONDITION #1;NTYPE=1 OR 3; INITIALIZE
R(1,1)=1.D0/KAB*PAB
R(2,1)=1.D0/KBC*PBC

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R(3,1)=1.D0/KCD*PCD
R(4,1)=1.D0/KAD*PAD
R(5,1)=1.D0/KAC*PAC
R(6,1)=1.D0/KBD*PBD
131 CONTINUE
IF (NTYPE .NE. 2) GO TO 132
C***CONDITION #1;NTYPE=2;INITIALIZE
R(1,1)=1.D0/KAB*PAB
R(2,1)=1.D0/KBC*PBC
R(3,1)=1.D0/KCD*PCD
R(4,1)=1.D0/KAD*PAD
R(7,1)=1.D0/KAE*PAE
R(8,1)=1.D0/KBE*PBE
R(9,1)=1.D0/KCE*PCE
R(10,1)=1.D0/KDE*PDE
132 CONTINUE
IF (NTYPE .NE. 3) GO TO 133
C***CONDITION #2;NTYPE=3;INITIALIZE
R(1,2)=1.D0/KAB2*PAB
R(2,2)=1.D0/KBC2*PBC
R(5,2)=1.D0/KAC2*PAC
R(7,2)=1.D0/KAE2*PAE
R(8,2)=1.D0/KBE2*PBE
R(9,2)=1.D0/KCE2*PCE
C***CONDITION #3;NTYPE=3;INITIALIZE
R(2,3)=1.D0/KBC3*PBC
R(3,3)=1.D0/KCD3*PCD
R(6,3)=1.D0/KBD3*PBD
R(8,3)=1.D0/KBE3*PBE
R(9,3)=1.D0/KCE3*PCE
R(10,3)=1.D0/KDE3*PDE
133 CONTINUE
C*** THIS SECTION SETS UP THE NORMAL EQUATIONS.
DO 500 L=1,NN
DO 500 J=1,NN
DO 500 I=1,NL
S(I)=R(I,L)*R(I,J)/P(I)
500 CC(L,J)=CC(L,J)+S(I)
C*** THIS SECTION CONSISTS OF A GAUSSIAN ELIMINATION PROCESS TO SOLVE
C*** THE NORMAL EQUATIONS FOR THE CORRELATES.
IF(N .NE. 1) GO TO 4
IF(CC(1,1) .EQ. 0.) GO TO 3
K(1)=W(1)/CC(1,1)
GO TO 202
3 GO TO 203
4 NLESS1=N-1
DO 13 I=1,NLESS1
BIG=ABS(CC(I,1))
L=I
IPLUS1=I+1
DO 6 J=IPLUS1,N
IF (ABS(CC(J,I)) .LE. BIG) GO TO 6
BIG=ABS(CC(J,I))
L=J
6 CONTINUE
IF (BIG .NE. 0.) GO TO 8
GO TO 203
8 IF (L .EQ. 1) GO TO 11
DO 10 J=1,N
TEMP=CC(L,J)
CC(L,J)=CC(I,J)
10 CC(I,J)=TEMP
TEMP=W(L)
W(L)=W(I)

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W(I)=TEMP
11 DO 13 J=IPLUS1,N
    QUOT=CC(J,I)/CC(I,I)
    DO 12 M=IPLUS1,N
12 CC(J,M)=CC(J,M)-QUOT*CC(I,M)
13 W(J)=W(J)-QUOT*W(I)
    IF (CC(N,N) .NE. 0.) GO TO 15
    GO TO 203
15 K(N)=W(N)/CC(N,N)
    I=N-1
16 SUM=0.
    IPLUS1=I+1
    DO 17 J=IPLUS1,N
17 SUM=SUM+CC(I,J)*K(J)
    K(I)=(W(I)-SUM)/CC(I,I)
    I=I-1
    IF (I .GT. 0) GO TO 16
202 WRITE (9,201)(K(I),I=1,N)
201 FORMAT (/, ' CORRELATES: ',/,3(D18.9,/),/)
    GO TO 205
203 WRITE (9,204)
204 FORMAT ( ' ***ERROR***=> IN CORRELATE MATRIX' )
    GO TO 999
205 CONTINUE
    SUMVV=0.
    DO 124 I=1,N
124 SUMVV=SUMVV+K(I)*WW(I)*1.0D12
C*** THIS SECTION COMPUTES THE CORRECTIONS TO EACH LINE IN MILLIMETERS.
    IF (NTYPE .EQ. 3) GO TO 130
    VAB=K(N)*PAB/KAB*10.**6
    VBC=K(N)*PBC/KBC*10.**6
    VCD=K(N)*PCD/KCD*10.**6
    VAD=K(N)*PAD/KAD*10.**6
    IF (NTYPE .EQ. 1) VAC=K(N)*PAC/KAC*10.**6
    IF (NTYPE .EQ. 1) VBD=K(N)*PBD/KBD*10.**6
    IF (NTYPE .NE. 2) GO TO 130
    VAE=K(N)*PAE/KAЕ*10.**6
    VBE=K(N)*PBE/KBE*10.**6
    VCE=K(N)*PCE/KCE*10.**6
    VDE=K(N)*PDE/KDE*10.**6
130 CONTINUE
    IF (NTYPE .NE. 3) GO TO 126
    VAB=PAB*(K(1)/KAB+K(2)/KAB2)*10.**6
    VBC=PBC*(K(1)/KBC+K(2)/KBC2+K(3)/KBC3)*10.**6
    VCD=PCD*(K(1)/KCD+K(3)/KCD3)*10.**6
    VAD=PAD*K(1)/KAD*10.**6
    VAC=PAC*(K(1)/KAC+K(2)/KAC2)*10.**6
    VBD=PBD*(K(1)/KBD+K(3)/KBD3)*10.**6
    VAE=PAE*K(2)/KAЕ2*10.**6
    VBE=PBE*(K(2)/KBE2+K(3)/KBE3)*10.**6
    VCE=PCE*(K(2)/KCE2+K(3)/KCE3)*10.**6
    VDE=PDE*K(3)/KDE3*10.**6
126 CONTINUE
    WRITE (9,127) VAB,VBC,VCD,VAD
127 FORMAT ( ' CORRECTIONS TO LINES (MTD: ',/, ' VAB=',F7.1,/,' VBC=',F7.
11./,' VCD=',F7.1,/,' VAD=',F7.1)
    IF (NTYPE .NE. 2) WRITE (9,128) VAC,VBD
128 FORMAT ( ' VAC=',F7.1,/,' VBD=',F7.1)
    IF (NTYPE .NE. 1) WRITE (9,129) VAE,VBE,VCE,VDE
129 FORMAT ( ' VAE=',F7.1,/,' VBE=',F7.1,/,' VCE=',F7.1,/,' VDE=',F7.1)
C*** THIS SECTION COMPUTES A CHECK ON SUM VV, USING THE LINE LENGTH
C*** CORRECTIONS, AND WRITES SUM VV, THE CHECK ON SUM VV, AND SIGMA.
    SUMVV2=VAB**2/PAB+VBC**2/PBC+VCD**2/PCD+VAD**2/PAD
    IF (NTYPE .NE. 2) SUMVV2=SUMVV2+VAC**2/PAC+VBD**2/PBD

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      IF (NTYPE .NE. 1) SUMVV2=SUMVV2+VAE**2/PAE+VBE**2/PBE+VCE**2/PCE+V
1DE**2/PDE
      ANU=DSQRT(SUMVV2/N)
      WRITE (9,138) SUMVV,SUMVV2,ANU
138 FORMAT (/, ' SUM VV=',F9.3,/, ' CHECK ON SUM VV=',F9.3,/, ' SIGMA=',F
17.3, ' ML/LINE')
C*** THIS SECTION ADDS THE CORRECTIONS TO EACH LINE.
      AB=AB+VAB*1.0D-6
      BC=BC+VBC*1.0D-6
      CD=CD+VCD*1.0D-6
      AD=AD+VAD*1.0D-6
      IF (NTYPE .NE. 2) AC=AC+VAC*1.0D-6
      IF (NTYPE .NE. 2) BD=BD+VBD*1.0D-6
      IF (NTYPE .EQ. 1) GO TO 134
      AE=AE+VAE*1.0D-6
      BE=BE+VBE*1.0D-6
      CE=CE+VCE*1.0D-6
      DE=DE+VDE*1.0D-6
134 CONTINUE
      WRITE (9,135) AB,BC,CD,AD
135 FORMAT (/, ' ADJUSTED LINE LENGTES (KMD:',/, ' AB=',F10.6,/, ' BC=',F
110.6,/, ' CD=',F10.6,/, ' AD=',F10.6)
      IF (NTYPE .NE. 2) WRITE (9,136) AC,BD
136 FORMAT ( ' AC=',F10.6,/, ' BD=',F10.6)
      IF (NTYPE .NE. 1) WRITE (9,137) AE,BE,CE,DE
137 FORMAT ( ' AE=',F10.6,/, ' BE=',F10.6,/, ' CE=',F10.6,/, ' DE=',F10.6)
C*** THIS SECTION COMPUTES A CHECK ON THE LINE LENGTH CORRECTIONS.
      CLOS=(VAB/KAB+VBC/KBC+VCD/KCD+VAD/KAD)*1.0D-6
      IF (NTYPE .NE. 2) CLOS=CLOS+(VAC/KAC+VBD/KBD)*1.0D-6
      IF (NTYPE .EQ. 2) CLOS=CLOS+(VAE/KAЕ+VBE/KBE+VCE/KCE+VDE/KDE)*1.0D-6
      CLOSE(1)=CLOS-W(1)
      IF (NTYPE .NE. 3) GO TO 141
      CLOS=(VAB/KAB2+VBC/KBC2+VAC/KAC2+VAE/KAЕ2+VBE/KBE2+VCE/KCE2)*1.D-6
      CLOSE(2)=CLOS-W(2)
      CLOS=(VBC/KBC3+VCD/KCD3+VBD/KBD3+VBE/KBE3+VCE/KCE3+VDE/KDE3)*1.D-6
      CLOSE(3)=CLOS-W(3)
141 CONTINUE
      WRITE (9,139)
139 FORMAT (/, ' CHECK OF CORRECTIONS:')
      WRITE (9,140) (I,CLOSE(I),I=1,N)
140 FORMAT (I3,':',D12.5)
C*** THIS SECTION GOES BACK AND RECOMPUTES EACH ANGLE, AND THE CONDI-
C*** TION RESIDUALS AFTER ADJUSTMENT.
      NO=1
      GO TO 104
144 CONTINUE
C*** THIS SECTION COMPUTES THE TRIANGLE CLOSURES AFTER ADJUSTMENT.
      IF (NTYPE .EQ. 2) GO TO 145
      Z(1)=(AQ(1,1)+AQ(1,2)+AQ(1,3)+AQ(1,4)-360.)*3600.
      Z(2)=(AQ(3,3)+AQ(2,4)-AQ(2,2)-AQ(3,1))*3600.
      Z(3)=(AQ(2,3)+AQ(3,2)-AQ(3,4)-AQ(2,1))*3600.
145 CONTINUE
      IF (NTYPE .EQ. 1) GO TO 146
      Z(4)=(AQ(6,1)+AQ(6,2)+AQ(6,3)+AQ(6,4)-360.)*3600.
      Z(5)=(AQ(5,1)+AQ(5,2)+AQ(5,3)+AQ(5,4)+AQ(4,1)+AQ(4,2)+AQ(4,3)+AQ(4
1,4)-360.)*3600.
146 CONTINUE
      WRITE (9,147)
147 FORMAT (/, ' TRIANGLE CONDITION CLOSURES:')
      II=4
      IF (NTYPE .NE. 2) II=1
      LL=5
      IF (NTYPE .EQ. 1) LL=3
      WRITE (9,148) (Z(I),I=II,LL)

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148 FORMAT (F9.3, ' ')
999 STOP
END
C*** FUNCTION ANGLE COMPUTES AN ANGLE, GIVEN THREE LINE LENGTHS BY
C*** USING THE COSINE LAW.
FUNCTION ANGLE(X, Y, Z)
DOUBLE PRECISION X, Y, Z, DACOS
ANGLE=DACOS((X**2+Y**2-Z**2)/(2.*X*Y))
RETURN
END
C*** FUNCTIONS KORM1, KORM2, AND KORM3 COMPUTE NORMALS TO THE LINES.
FUNCTION KORM1(V, W, X, Y, Z)
DOUBLE PRECISION V, W, X, Y, Z, DCOS, DSIN
KORM1=1.D0/((W*DCOS(Y)-V)/(V*W*DSIN(Y))+(X*DCOS(Z)-V)/(V*X*DSIN(Z)
1))
RETURN
END
FUNCTION KORM2(T, U, V, W, X, Y, Z)
DOUBLE PRECISION T, U, V, W, X, Y, Z, DSIN
KORM2=1.D0/(T/(U*V*DSIN(W))+T/(X*Y*DSIN(Z)))
RETURN
END
FUNCTION KORM3(W, X, Y, Z)
DOUBLE PRECISION W, X, Y, Z, DSIN
KORM3=1.D0/(W/(X*Y*DSIN(Z)))
RETURN
END
C*** FUNCTION WEICHT COMPUTES THE WEIGHT OF A LINE, GIVEN ITS LENGTH.
C*** USING A DESIRED WEIGHTING FUNCTION.
FUNCTION WEICHT(D)
DOUBLE PRECISION D
IF (D .EQ. 0.0) WEIGHT=0.0
IF (D .EQ. 0.0) RETURN
WEICHT=D/D
RETURN
END

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## ADJUSTMENT OF LUKE QUADRILATERAL WITH DIAGONALS AND CENTER POINT ARPA=3034.34

## MEASURED LINE LENGTHS:

AB= 31.251588  
 AC= 28.768443  
 AD= 3.272860  
 AE= 20.686783  
 BC= 7.214161  
 BD= 30.131599  
 BE= 10.871607  
 CD= 26.983473  
 CE= 8.641175  
 DE= 19.332759

## CALCULATED ANGLES (RADIAN):

GAMMA 1-4:  
   0.117186247E+01  
   0.110856060E+01  
   0.190515144E+01  
   0.209760642E+01

ALPHA 1-4:  
   0.945486288E+00  
   0.100257309E+00  
   0.180665298E+01  
   0.228136919E+00

BETA 1-4:  
   0.226379077E+00  
   0.100330429E+01  
   0.984999493E-01  
   0.186947268E+01

DELTA 1-4:  
   0.107096239E+01  
   0.192871624E+00  
   0.150238923E+01  
   0.176104279E+00

THETA 1-4:  
   0.100904448E+00  
   0.915684468E+00  
   0.402763160E+00  
   0.192149787E+01

PHI 1-4:  
   0.234781658E+01  
   0.723518951E+00  
   0.256272521E+01  
   0.149132392E+00

## CALCULATED ANGLES (DEGREES):

GAMMA 1-4:  
   67 8 33.985  
   63 30 57.037  
   109 9 25.693  
   120 11 2.381

ALPHA 1-4:  
   54 10 20.546  
   5 44 39.596  
   103 30 48.927  
   13 4 16.617

BETA 1-4:  
   12 58 14.036  
   57 46 17.689  
   5 38 37.073  
   107 6 46.419

DELTA 1-4:  
   61 21 41.850  
   11 3 2.628  
   86 4 50.024  
   10 5 24.115

THETA 1-4:  
   5 46 53.036  
   52 27 53.479  
   23 4 35.865  
   110 5 37.386

PHI 1-4:  
   163 10 4.335  
   41 27 16.496  
   146 50 0.020  
   8 32 40.764

## ANGLE CONDITION CLOSURES (RADIAN):

1: -0.4382E-05 = -0.904"  
 2: 0.1221E-04 = 2.518"  
 3: -0.5772E-05 = -1.190"

## NORMALS TO LINES (KMD):

KAB= 2.188100  
 KBC= 18.493254  
 KCD= 1.870416  
 KAD= 7.904133  
 KAC= -1.925249  
 KBD= -2.067996  
 KAB2= -2.083349  
 KBC2= -8.620965

KAC2= -2.588784  
 KAE2= 1.161865  
 KBE2= 1.846198  
 KCE2= 2.656267  
 KBC3= -8.620965  
 KBD3= -1.005518  
 KCD3= -3.387011  
 KBE3= 0.942579  
 KCE3= 3.577188  
 KDE3= 0.778910

INVERSE WEIGHTS OF LINES:

PAB= 1.000  
 PBC= 1.000  
 PCD= 1.000  
 PAD= 1.000  
 PAC= 1.000  
 PBD= 1.000  
 PAE= 1.000  
 PBE= 1.000  
 PCE= 1.000  
 PDE= 1.000

CORRELATES:

-0.322236737E-05  
 0.895756309E-05  
 -0.278095789E-05

CORRECTIONS TO LINES (MM):

VAB= -5.8  
 VBC= -0.9  
 VCD= -0.9  
 VAD= -0.4  
 VAC= -1.8  
 VBD= 4.3  
 VAE= 7.7  
 VBE= 1.9  
 VCE= 2.6  
 VDE= -3.6

SUM VV= 139.502  
 CHECK ON SUM VV= 139.502  
 SIGMA= 6.819 MM/LINE

ADJUSTED LINE LENGTHS (KM):

AB= 31.251582  
 BC= 7.214160  
 CD= 26.983472  
 AD= 3.272860  
 AC= 28.768441  
 BD= 30.131603  
 AE= 20.686796  
 BE= 10.871609  
 CE= 8.641178  
 DE= 19.332755

CHECK OF CORRECTIONS:

1: -0.27756E-16  
 2: 0.10747E-06  
 3: 0.40439E-05

ADJUSTED ANGLES (RADIAN):

ADJUSTED ANGLES (DEGREES):

GAMMA 1-4:		GAMMA 1-4:	
0.117186565E+01		67	8 34.641
0.110856111E+01		63	30 57.143
0.190515237E+01		109	9 25.885
0.209760617E+01		120	11 2.331
ALPHA 1-4:		ALPHA 1-4:	
0.945486525E+00		54	10 20.595
0.100257617E+00		5	44 39.618
0.180665242E+01		103	30 48.811
0.228136782E+00		13	4 16.589
BETA 1-4:		BETA 1-4:	
0.226379122E+00		12	58 14.046
0.100830350E+01		57	46 17.525
0.984999572E-01		5	38 37.075
0.186946939E+01		107	6 45.741
DELTA 1-4:		DELTA 1-4:	
0.107095870E+01		61	21 41.090
0.192876475E+00		11	3 3.629
0.150238932E+01		86	4 50.042
0.176104320E+00		10	5 24.123
THETA 1-4:		THETA 1-4:	
0.100906943E+00		5	46 53.551
0.915684639E+00		52	27 53.515
0.402763052E+00		23	4 35.843
0.192150185E+01		110	5 38.207
PHI 1-4:		PHI 1-4:	
0.284780924E+01		163	10 2.820
0.723518692E+00		41	27 16.443
0.256272528E+01		146	50 0.034
0.149132098E+00		8	32 40.703

ANGLE CONDITION CLOSURES AFTER ADJUSTMENT (RAD):

1: 0.2910E-10 = 0.000"  
 2: 0.1164E-09 = 0.000"  
 3: 0.2037E-09 = 0.000"

TRIANGLE CONDITION CLOSURES:

0.000"  
 0.000"  
 0.000"  
 0.000"  
 0.000"





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C      PUTED BY USE OF THE COSINE LAW. IF NINPUT=2 THE REST OF THE DATA C
C      CARDS WILL EACH CONTAIN ONE ANGLE MEASUREMENT IN DEGREES, MIN- C
C      UTES, AND SECONDS. COLUMNS 1-3 MUST CONTAIN DEGREES; COLUMNS 5-6 C
C      CONTAIN MINUTES; WHILE COLUMNS 8-13 CONTAIN SECONDS IN AN F6.3 C
C      FORMAT. THE ANGLES MUST BE INPUT IN THE FOLLOWING ORDER: ALPHA C
C      1-4, BETA 1-4, GAMMA 1-4, THETA 1-4, DELTA 1-4, AND PHI 1-4. IN C
C      THE PROGRAM "N" IS THE NUMBER OF CONDITIONS OF THE ADJUSTMENT. C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C*** THIS SECTION INITIALIZES VARIABLES AND READS THE INPUT.
      DIMENSION KAD(6,4),KAM(6,4),XADJ(12)
      DOUBLE PRECISION DX,PI,OT,OTH,OF,OFV,TTH,TF,TFV,THF,THFV,FFV,A(6,4
1),AM(6,4),AS(6,4),DACOS,AR(6,4),W(10),BIG,SUM,DSIN,DLOG10,DTAN,
2ALPHA(6,4),R(24,10),S(24),C(10,10),Q(10),TEMP,QUOT,SUMVV,AMU,
3DSQRT,DN,WW(10),V(24),VV,VA(6,4),SUMVV2,G1
      READ (8,100) XADJ
100  FORMAT (12A6)
      READ (8,160) NTYPE
160  FORMAT (T7,11)
      READ (8,132) NINPUT
132  FORMAT (T8,11)
      N1=4
      N2=6
      IF (NINPUT .EQ. 1) N1=3
      IF (NINPUT .EQ. 1) N2=2
      N3=N1+N2
      IF (NTYPE .EQ. 1) N=N1
      IF (NTYPE .EQ. 2) N=N2
      IF (NTYPE .EQ. 3) N=N3
      DX=0.43429D00/206265.D00
      PI=3.1415926535898D00
      NO=0
      DN=DFLOAT(N)
C*** NSID = NUMBER OF SIDES OF FIGURE BEING ADJUSTED
C*** NAN = NUMBER OF DIFFERENT TYPES OF ANGLES IN FIGURE BEING ADJUSTED;
C*** I.E. ALPHAS, BETAS, GAMMAS, ETC.
      NAN=6
      NNN=1
      NSID=4
      IF (NTYPE .EQ. 1) NAN=3
      IF (NTYPE .EQ. 2) NNN=4
      NAXNS=NAN*NSID
      WRITE (9,100) XADJ
      WRITE (9,124) N
124  FORMAT (' NUMBER OF CONDITIONS=',I3,/)
      IF (NINPUT .EQ. 2) GO TO 133
      READ (8,101) OT,OTH,OF,OFV,TTH,TF,TFV
101  FORMAT (7F10.3)
      READ (8,102) THF,THFV,FFV
102  FORMAT (3F10.3)
      WRITE (9,103) OT,OTH,OF,OFV,TTH,TF,TFV,THF,THFV,FFV
103  FORMAT (' LENGTH OF LINES:',/, ' 1-2=',F10.3,/, ' 1-3=',F10.3,/, ' 1-
14=',F10.3,/, ' 1-5=',F10.3,/, ' 2-3=',F10.3,/, ' 2-4=',F10.3,/, ' 2-5=
2',F10.3,/, ' 3-4=',F10.3,/, ' 3-5=',F10.3,/, ' 4-5=',F10.3)
C*** THIS SECTION COMPUTES EACH ANGLE OF THE FIGURE IN RADIANS AND IN
C*** DEGREES, MINUTES, AND SECONDS, BY USE OF THE COSINE LAW.
      IF (NTYPE .EQ. 2) GO TO 508
      AR(1,1)=DACOS((OTH**2+OF**2-THF**2)/(2.*OTH*OF))
      AR(1,2)=DACOS((OT**2+TTH**2-OTH**2)/(2.*OT*TTH))
      AR(1,3)=DACOS((TTH**2+THF**2-TF**2)/(2.*TTH*THF))
      AR(1,4)=DACOS((OF**2+TF**2-OT**2)/(2.*OF*TF))
      AR(2,1)=DACOS((OT**2+OF**2-TF**2)/(2.*OT*OF))
      AR(2,2)=DACOS((TTH**2+TF**2-THF**2)/(2.*TTH*TF))
      AR(2,3)=DACOS((OTH**2+TTH**2-OT**2)/(2.*OTH*TTH))

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AR(2,4)=DACOS((OF**2+THF**2-OTH**2)/(2.*OF*THF))
AR(3,1)=DACOS((OT**2+OTH**2-TTH**2)/(2.*OT*OTH))
AR(3,2)=DACOS((OT**2+TF**2-OF**2)/(2.*OT*TF))
AR(3,3)=DACOS((OTH**2+THF**2-OF**2)/(2.*OTH*THF))
AR(3,4)=DACOS((TF**2+THF**2-TTH**2)/(2.*TF*THF))
IF (NTYPE .EQ. 1) GO TO 106
508 CONTINUE
AR(4,1)=DACOS((OT**2+OFV**2-TFV**2)/(2.*OT*OFV))
AR(4,2)=DACOS((TF**2+TFV**2-FFV**2)/(2.*TF*TFV))
AR(4,3)=DACOS((OTH**2+THFV**2-OFV**2)/(2.*OTH*THFV))
AR(4,4)=DACOS((THF**2+FFV**2-THFV**2)/(2.*THF*FFV))
AR(5,1)=DACOS((OFV**2+OTH**2-THFV**2)/(2.*OFV*OTH))
AR(5,2)=DACOS((OT**2+TFV**2-OFV**2)/(2.*OT*TFV))
AR(5,3)=DACOS((THF**2+THFV**2-FFV**2)/(2.*THF*THFV))
AR(5,4)=DACOS((TF**2+FFV**2-TFV**2)/(2.*TF*FFV))
AR(6,1)=DACOS((OFV**2+TFV**2-OT**2)/(2.*OFV*TFV))
AR(6,2)=DACOS((TFV**2+FFV**2-TF**2)/(2.*TFV*FFV))
AR(6,3)=DACOS((OFV**2+THFV**2-OTH**2)/(2.*OFV*THFV))
AR(6,4)=DACOS((THFV**2+FFV**2-THF**2)/(2.*THFV*FFV))
106 DO 105 I=NNN,NAN
DO 105 J=1,NSID
A(I,J)=AR(I,J)*180./PI
KAD(I,J)=A(I,J)
AM(I,J)=(A(I,J)-KAD(I,J))*60.
KAM(I,J)=AM(I,J)
105 AS(I,J)=(AM(I,J)-KAM(I,J))*60.
IF (NO .EQ. 1) GO TO 199
147 CONTINUE
IF (NTYPE .EQ. 1) GO TO 506
IF (NTYPE .EQ. 2) GO TO 509
WRITE (9,104) ((AR(I,J),KAD(I,J),KAM(I,J),AS(I,J),J=1,NSID),I=NNN,
1NAN)
104 FORMAT (' "OBSERVED" ANGLES (RADIANS):',T40,' "OBSERVED" ANGLES (DE
1GREES):',/,', ALPHA 1-4:',T40,' ALPHA 1-4:',/,4(D20.9,T40,I4,1X,I3,1
2X,F7.3,/), ' BETA 1-4:',T40,' BETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F
37.3,/), ' GAMMA 1-4:',T40,' GAMMA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7
4.3,/), ' THETA 1-4:',T40,' THETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.
53,/), ' DELTA 1-4:',T40,' DELTA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3
6,/), ' PHI 1-4:',T40,' PHI 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3,/))
GO TO 108
506 WRITE (9,125) ((AR(I,J),KAD(I,J),KAM(I,J),AS(I,J),J=1,NSID),I=NNN,
1NAN)
125 FORMAT (' "OBSERVED" ANGLES (RADIANS):',T40,' "OBSERVED" ANGLES (DE
1GREES):',/,', ALPHA 1-4:',T40,' ALPHA 1-4:',/,4(D20.9,T40,I4,1X,I3,1
2X,F7.3,/), ' BETA 1-4:',T40,' BETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F
37.3,/), ' GAMMA 1-4:',T40,' GAMMA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7
4.3,/))
GO TO 108
509 WRITE (9,126) ((AR(I,J),KAD(I,J),KAM(I,J),AS(I,J),J=1,NSID),I=NNN,
1NAN)
126 FORMAT (' "OBSERVED" ANGLES (RADIANS):',T40,' "OBSERVED" ANGLES (DE
1GREES):',/,', THETA 1-4:',T40,' THETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1
2X,F7.3,/), ' DELTA 1-4:',T40,' DELTA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X
3,F7.3,/), ' PHI 1-4:',T40,' PHI 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3
4,/))
108 CONTINUE
GO TO 134
133 CONTINUE
READ (8,135) ((KAD(I,J),KAM(I,J),AS(I,J),J=1,NSID),I=NNN,NAN)
135 FORMAT (I3,1X,I2,1X,F6.3)
DO 146 I=NNN,NAN
DO 146 J=1,NSID
146 A(I,J)=AS(I,J)/3600.+KAM(I,J)/60.+KAD(I,J)
DO 136 I=NNN,NAN

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DO 136 J=1,NSID
136 AR(I,J)=A(I,J)*PI/180.
GO TO 147
134 CONTINUE
C*** THIS SECTION CONTAINS THE CONDITIONS, AND COMPUTES THE RESIDUALS
C*** BEFORE ADJUSTMENT IN SECONDS, EXCEPT FOR SIDE CONDITIONS.
IF (NINPUT .EQ. 1) GO TO 137
IF (NTYPE .EQ. 2) GO TO 511
W(1)=(A(3,1)+A(3,2)+A(3,3)+A(3,4)-360.)*3600.
W(2)=(A(2,4)+A(1,3)-A(1,2)-A(2,1))*3600.
W(3)=(A(1,4)+A(2,2)-A(2,3)-A(1,1))*3600.
W(4)=DLOG10(DSIN(AR(1,1))*DSIN(AR(1,2))*DSIN(AR(1,3))*DSIN(AR(1,4)
1)/DSIN(AR(2,1))/DSIN(AR(2,2))/DSIN(AR(2,3))/DSIN(AR(2,4)))*10.**6
IF (NTYPE .EQ. 1) GO TO 507
511 CONTINUE
W(5)=(A(4,1)+A(6,1)+A(5,2)-180.)*3600.
W(6)=(A(4,2)+A(6,2)+A(5,4)-180.)*3600.
W(7)=(A(4,3)+A(6,3)+A(5,1)-180.)*3600.
W(8)=(A(4,4)+A(6,4)+A(5,3)-180.)*3600.
W(9)=(A(6,1)+A(6,2)+A(6,3)+A(6,4)-360.)*3600.
W(10)=DLOG10(DSIN(AR(5,1))*DSIN(AR(5,2))*DSIN(AR(5,3))*DSIN(AR(5,4)
1)/DSIN(AR(4,1))/DSIN(AR(4,2))/DSIN(AR(4,3))/DSIN(AR(4,4)))*10.**6
IF (NTYPE .NE. 2) GO TO 507
GO TO 139
137 CONTINUE
IF (NTYPE .EQ. 2) GO TO 138
W(1)=(A(3,1)+A(3,2)+A(3,3)+A(3,4)-360.)*3600.
W(2)=(A(2,4)+A(1,3)-A(1,2)-A(2,1))*3600.
W(3)=(A(1,4)+A(2,2)-A(2,3)-A(1,1))*3600.
IF (NTYPE .EQ. 1) GO TO 507
138 CONTINUE
W(4)=(A(6,1)+A(6,2)+A(6,3)+A(6,4)-360.)*3600.
W(5)=(A(4,1)+A(4,2)+A(4,3)+A(4,4)+A(5,1)+A(5,2)+A(5,3)+A(5,4)-360.
1)*3600.
139 CONTINUE
IF (NTYPE .NE. 2) GO TO 507
DO 510 I=1,N
510 W(I)=W(I+N1)
507 CONTINUE
IF (NO .EQ. 1) GO TO 198
DO 150 I=1,N
150 WW(I)=W(I)
WRITE (9,109)(W(I),I=1,N)
109 FORMAT (' CONDITION RESIDUALS:',/, ' I:',F12.5,/, ' II:',F12.5,/, ' I
111:',F12.5,/, ' IV:',F12.5,/, ' V:',F12.5,/, ' VI:',F12.5,/, ' VII:',F
212.5,/, ' VIII:',F12.5,/, ' IX:',F12.5,/, ' X:',F12.5,/)
C*** THIS SECTION COMPUTES THE LOG SINE DIFFERENCE OF EACH ANGLE USING
C*** THE FORMULA: LSD=M/RHO*COTAN(ANGLE).
DO 110 I=NNN,NAN
DO 110 J=1,NSID
110 ALPHA(I,J)=DX*(1./DTAN(AR(I,J)))*10.**6
C*** THIS SECTION INITIALIZES THE CORRELATE MATRIX.
DO 112 I=1,NAXNS
DO 112 J=1,10
112 R(I,J)=0.
IF (NINPUT .EQ. 1) GO TO 140
IF (NTYPE .EQ. 2) GO TO 512
C*** THIS SECTION INITIALIZES THE MATRIX FOR ANGLE INPUT
C***CONDITION #1; INITIALIZE
DO 115 I=9,12
115 R(I,1)=1.
C***CONDITION #2; INITIALIZE
R(8,2)=1.
R(3,2)=1.

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      R(2,2)=-1.
      R(5,2)=-1.
C***CONDITION #3; INITIALIZE
      R(4,3)=1.
      R(6,3)=1.
      R(7,3)=-1.
      R(1,3)=-1.
C***CONDITION #4; INITIALIZE
      R(1,4)= ALPHA(1,1)
      R(2,4)= ALPHA(1,2)
      R(3,4)= ALPHA(1,3)
      R(4,4)= ALPHA(1,4)
      R(5,4)=-ALPHA(2,1)
      R(6,4)=-ALPHA(2,2)
      R(7,4)=-ALPHA(2,3)
      R(8,4)=-ALPHA(2,4)
      IF (NTYPE .EQ. 1) GO TO 513
512 CONTINUE
C***CONDITION #5; INITIALIZE
      R(13,5)=1.
      R(18,5)=1.
      R(21,5)=1.
C***CONDITION #6; INITIALIZE
      R(14,6)=1.
      R(22,6)=1.
      R(20,6)=1.
C***CONDITION #7; INITIALIZE
      R(15,7)=1.
      R(17,7)=1.
      R(23,7)=1.
C***CONDITION #8; INITIALIZE
      R(16,8)=1.
      R(19,8)=1.
      R(24,8)=1.
C***CONDITION #9; INITIALIZE
      DO 114 I=21,24
114 R(I,9)=1.
C***CONDITION #10; INITIALIZE
      R(17,10)=ALPHA(5,1)
      R(18,10)=ALPHA(5,2)
      R(19,10)=ALPHA(5,3)
      R(20,10)=ALPHA(5,4)
      R(13,10)=-ALPHA(4,1)
      R(14,10)=-ALPHA(4,2)
      R(15,10)=-ALPHA(4,3)
      R(16,10)=-ALPHA(4,4)
      IF (NTYPE .NE. 2) GO TO 513
      GO TO 141
140 CONTINUE
      IF (NTYPE .EQ.2) GO TO 142
C*** THIS SECTION INITIALIZES THE MATRIX FOR LINE LENGTH INPUT
C***CONDITION #1; INITIALIZE
      DO 143 I=9,12
143 R(I,1)=1.
C***CONDITION #2; INITIALIZE
      R(8,2)=1.
      R(3,2)=1.
      R(2,2)=-1.
      R(5,2)=-1.
C***CONDITION #3; INITIALIZE
      R(4,3)=1.
      R(6,3)=1.
      R(7,3)=-1.
      R(1,3)=-1.

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      IF (NTYPE .EQ. 1) GO TO 513
142 CONTINUE
C***CONDITION #4; INITIALIZE
      DO 144 I=21,24
144 R(I,4)=1.
C***CONDITION #5; INITIALIZE
      DO 145 I=13,20
145 R(I,5)=1.
      IF (NTYPE .NE. 2) GO TO 513
141 CONTINUE
      DO 514 I=1,NAXNS
      DO 514 J=1,N
514 R(I,J)=R(I,J+N1)
513 CONTINUE
C*** THIS SECTION SETS UP THE NORMAL EQUATIONS.
      DO 118 II=1,N
      DO 118 JJ=1,N
118 C(II,JJ)=0.
      DO 500 K=1,N
      DO 500 J=1,N
      DO 500 I=1,NAXNS
      S(I)=R(I,K)*R(I,J)
500 C(K,J)=C(K,J)+S(I)
C*** THIS SECTION CONSISTS OF A GAUSSIAN ELIMINATION PROCESS TO SOLVE
C*** THE NORMAL EQUATIONS FOR THE CORRELATES.
      IF(N .NE. 1) GO TO 4
      IF(C(1,1) .EQ. 0.) GO TO 3
      Q(1)=W(1)/C(1,1)*(-1.)
      GO TO 202
3 GO TO 203
4 NLESS1=N-1
      DO 13 I=1,NLESS1
      BIG=ABS(C(I,I))
      L=I
      IPLUS1=I+1
      DO 6 J=IPLUS1,N
      IF (ABS(C(J,I)) .LE. BIG) GO TO 6
      BIG=ABS(C(J,I))
      L=J
6 CONTINUE
      IF (BIG .NE. 0.) GO TO 8
      GO TO 203
8 IF (L .EQ. 1) GO TO 11
      DO 10 J=1,N
      TEMP=C(L,J)
      C(L,J)=C(I,J)
10 C(I,J)=TEMP
      TEMP=W(L)
      W(L)=W(I)
      W(I)=TEMP
11 DO 13 J=IPLUS1,N
      QUOT=C(J,I)/C(I,I)
      DO 12 K=IPLUS1,N
12 C(J,K)=C(J,K)-QUOT*C(I,K)
13 W(J)=W(J)-QUOT*W(I)
      IF (C(N,N) .NE. 0.) GO TO 15
      GO TO 203
15 Q(N)=W(N)/C(N,N)
      I=N-1
16 SUM=0.
      IPLUS1=I+1
      DO 17 J=IPLUS1,N
17 SUM=SUM+C(I,J)*Q(J)
      Q(I)=(W(I)-SUM)/C(I,I)

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      I=I-1
      IF (I .GT. 0) GO TO 16
      DO 18 KK=1,N
18     Q(KK)=-Q(KK)
202    WRITE (9,201)(Q(I),I=1,N)
201    FORMAT (' CORRELATES:',/,13(F15.3,/,/),/)
      GO TO 205
203    WRITE (9,204)
204    FORMAT (' ***ERROR***=> IN CORRELATE MATRIX')
      GO TO 999
C*** THIS SECTION COMPUTES SUM VV AND MU
205    SUMVV=0.
      DO 119 I=1,N
119    SUMVV=SUMVV-Q(I)*WW(I)
C*** THIS SECTION COMPUTES THE CORRECTIONS TO EACH ANGLE IN SECONDS.
      DO 502 I=1,NAXNS
502    V(I)=0.
      DO 501 I=1,NAXNS
      DO 501 J=1,N
      VV=R(I,J)*Q(J)
501    V(I)=V(I)+VV
      M=0
      IF (NTYPE .EQ. 2) M=12
      DO 504 I=NNN,NAN
      DO 504 J=1,NSID
      M=M+1
504    VA(I,J)=V(M)
      IF (NTYPE .EQ. 1) GO TO 702
      IF (NTYPE .EQ. 2) GO TO 703
      WRITE (9,121) ((VA(I,J),I=NNN,NAN),J=1,NSID)
121    FORMAT (' CORRECTIONS TO ANGLES:',/, ' ALPHA 1-4:',6X, ' BETA 1-4:',
18X, ' GAMMA 1-4:',7X, ' THETA 1-4:',7X, ' DELTA 1-4:',5X, ' PHI 1-4:',/,
24(4X,F7.3, ' ',5(9X,F7.3, ' ')),/,/)
      GO TO 701
702    WRITE (9,130) ((VA(I,J),I=NNN,NAN),J=1,NSID)
130    FORMAT (' CORRECTIONS TO ANGLES:',/, ' ALPHA 1-4:',6X, ' BETA 1-4:',
18X, ' GAMMA 1-4:',7X,/,12(4X,F7.3, ' ',2(9X,F7.3, ' ')),/,/)
      GO TO 701
703    WRITE (9,131) ((VA(I,J),I=NNN,NAN),J=1,NSID)
131    FORMAT (' CORRECTIONS TO ANGLES:',/, ' THETA 1-4:',7X, ' DELTA 1-4:',
1,5X, ' PHI 1-4:',/,12(4X,F7.3, ' ',2(9X,F7.3, ' ')),/,/)
701    CONTINUE
C*** THIS SECTION COMPUTES A CHECK ON SUM VV, USING THE ANGLE
C*** CORRECTIONS, AND WRITES SUM VV, THE CHECK ON SUM VV, AND MU.
      SUMVV2=0.D00
      DO 515 I=NNN,NAN
      DO 515 J=1,NSID
      GI=VA(I,J)**2
515    SUMVV2=SUMVV2+GI
      AMU=DSQRT(SUMVV2/DN)
      WRITE (9,120) SUMVV,SUMVV2,AMU
120    FORMAT (' SUM VV=',F10.3,/, ' CHECK ON SUM VV=',F10.3,/, ' MU=',F10.
13,/)
C*** THIS SECTION ADDS CORRECTIONS TO EACH ANGLE.
      DO 503 I=NNN,NAN
      DO 503 J=1,NSID
503    AR(I,J)=AR(I,J)+VA(I,J)/648000.*PI
      NO=NO+1
      GO TO 106
199    CONTINUE
      IF (NTYPE .EQ. 1) GO TO 606
      IF (NTYPE .EQ. 2) GO TO 609
      WRITE (9,127) ((AR(I,J),KAD(I,J),KAM(I,J),AS(I,J),J=1,NSID),I=NNN,
1NAN)

```

```

127 FORMAT (' "ADJUSTED" ANGLES (RADIAN):',T40,' "ADJUSTED" ANGLES (DE
1GREES):',/,,' ALPHA 1-4:',T40,' ALPHA 1-4:',/,4(D20.9,T40,I4,1X,I3,1
2X,F7.3,/,), ' BETA 1-4:',T40,' BETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F
37.3,/,), ' GAMMA 1-4:',T40,' GAMMA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7
4.3,/,), ' THETA 1-4:',T40,' THETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.
53,/,), ' DELTA 1-4:',T40,' DELTA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3
6,/,), ' PHI 1-4:',T40,' PHI 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3,/,))
GO TO 700
606 WRITE (9,128) ((AR(I,J),KAD(I,J),KAM(I,J),AS(I,J),J=1,NSID),I=NNN,
1NAN)
128 FORMAT (' "ADJUSTED" ANGLES (RADIAN):',T40,' "ADJUSTED" ANGLES (DE
1GREES):',/,,' ALPHA 1-4:',T40,' ALPHA 1-4:',/,4(D20.9,T40,I4,1X,I3,1
2X,F7.3,/,), ' BETA 1-4:',T40,' BETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F
37.3,/,), ' GAMMA 1-4:',T40,' GAMMA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7
4.3,/,))
GO TO 700
609 WRITE (9,129) ((AR(I,J),KAD(I,J),KAM(I,J),AS(I,J),J=1,NSID),I=NNN,
1NAN)
129 FORMAT (' "ADJUSTED" ANGLES (RADIAN):',T40,' "ADJUSTED" ANGLES (DE
1GREES):',/,,' THETA 1-4:',T40,' THETA 1-4:',/,4(D20.9,T40,I4,1X,I3,1
2X,F7.3,/,), ' DELTA 1-4:',T40,' DELTA 1-4:',/,4(D20.9,T40,I4,1X,I3,1X
3,F7.3,/,), ' PHI 1-4:',T40,' PHI 1-4:',/,4(D20.9,T40,I4,1X,I3,1X,F7.3
4,/,))
700 CONTINUE
C*** THIS SECTION COMPUTES THE CONDITION RESIDUALS AFTER ADJUSTMENT.
GO TO 108
198 WRITE (9,123)(W(I),I=1,N)
123 FORMAT (' CONDITION RESIDUALS AFTER ADJUSTMENT:',/,,' I:',F12.5,/,,'
1 II:',F12.5,/,,' III:',F12.5,/,,' IV:',F12.5,/,,' V:',F12.5,/,,' VI:',
2F12.5,/,,' VII:',F12.5,/,,' VIII:',F12.5,/,,' IX:',F12.5,/,,' X:',F12.
35,/)
999 STOP
END

```

ADJUSTMENT OF LUKE QUADRILATERAL WITH DIAGONALS ONLY  
 NUMBER OF CONDITIONS= 3

LENGTH OF LINES:

1-2= 31251.588  
 1-3= 3272.860  
 1-4= 28768.443  
 1-5= 20686.788  
 2-3= 30131.599  
 2-4= 7214.161  
 2-5= 10871.607  
 3-4= 26983.473  
 3-5= 19332.759  
 4-5= 8641.175

"OBSERVED" ANGLES (RADIAN):

ALPHA 1-4:  
 0.945486288E+00  
 0.100257509E+00  
 0.228136919E+00  
 0.180665298E+01  
 BETA 1-4:  
 0.226379077E+00  
 0.100830429E+01  
 0.186947268E+01  
 0.984999493E-01  
 GAMMA 1-4:  
 0.117186247E+01  
 0.110856060E+01  
 0.209760642E+01  
 0.190515144E+01

"OBSERVED" ANGLES (DEGREES):

ALPHA 1-4:  
 54 10 20.546  
 5 44 39.596  
 13 4 16.617  
 103 30 48.927  
 BETA 1-4:  
 12 58 14.036  
 57 46 17.689  
 107 6 46.419  
 5 38 37.073  
 GAMMA 1-4:  
 67 8 33.985  
 63 30 57.037  
 120 11 2.381  
 109 9 25.693

CONDITION RESIDUALS:

I: -0.90391  
 II: 0.05821  
 III: -0.34951  
 IV:

CORRELATES:

0.226  
 -0.015  
 0.087

CORRECTIONS TO ANGLES:

ALPHA 1-4:	BETA 1-4:	GAMMA 1-4:
-0.087"	0.015"	0.226"
0.015"	0.087"	0.226"
-0.015"	-0.087"	0.226"
0.087"	-0.015"	0.226"

SUM VV= 0.236  
 CHECK ON SUM VV= 0.236  
 MU= 0.280

"ADJUSTED" ANGLES (RADIAN):

ALPHA 1-4:  
 0.945485864E+00  
 0.100257580E+00  
 0.228136849E+00  
 0.180665340E+01  
 BETA 1-4:  
 0.226379148E+00  
 0.100830471E+01  
 0.186947225E+01  
 0.984998788E-01

"ADJUSTED" ANGLES (DEGREES):

ALPHA 1-4:  
 54 10 20.459  
 5 44 39.610  
 13 4 16.603  
 103 30 49.014  
 BETA 1-4:  
 12 58 14.051  
 57 46 17.776  
 107 6 46.332  
 5 38 37.058



GAMMA 1-4:

0.117186356E+01  
0.110856169E+01  
0.209760751E+01  
0.190515254E+01

GAMMA 1-4:

67	8	34.211
63	30	57.263
120	11	2.607
109	9	25.919

CONDITION RESIDUALS AFTER ADJUSTMENT:

I: 0.00000  
II: 0.00000  
III: 0.00000  
IV:

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C          ***** GEOG *****
C          PROGRAM TO COMPUTE HTM PLANE, GEOGRAPHICAL, AND USR COORDIN-
C          ATES OF A POINT; WRITTEN BY BRUCE SCHENCK, APRIL, 1978.
C
C          PROGRAM COMPUTES PLANE COORDINATES OF UNKNOWN POINT FROM
C          BASELINE OF TWO KNOWN POINTS AND DISTANCES FROM EACH OF THESE
C          POINTS TO UNKNOWN POINT. IF ONLY ONE POINT OF BASELINE IS KNOWN,
C          APPROXIMATE COORDINATES OF SECOND BASELINE POINT ARE SUPPLIED,
C          AND THE AZIMUTH OF THE BASELINE IS COMPUTED. BY USING THIS AZI-
C          MUTH AND THE KNOWN DISTANCE, EXACT COORDINATES OF THE SECOND
C          BASELINE POINT IN THE SYSTEM ARE COMPUTED, AND THE PROGRAM PRO-
C          CEEDS BY USING THESE COORDINATES.
C          INPUT TO PROGRAM IS AS FOLLOWS:
C          CARD #1: HEADER CARD WHICH WILL BE PRINTED AT TOP OF OUTPUT.
C          CARD #2: COLS. 1-16; BASE POINT #1
C                   COLS. 17-32; BASE POINT #2
C                   COLS. 33-48; UNKNOWN POINT
C          CARD #3: COL. 7; "NTYPE", IF NTYPE=1, COORDINATES OF BASE
C          POINT #2 ARE APPROXIMATE; IF NTYPE=2, COORDINATES OF BASE POINT
C          #2 ARE KNOWN, AND WILL NOT BE COMPUTED.
C                   COL. 16; "ISIGNY", + IF UNKNOWN POINT IS ABOVE BASE-
C          LINE, - IF UNKNOWN POINT IS BELOW BASELINE.
C          CARD #4: COLS. 1-10; "DBL", HTM PLANE DISTANCE FROM BASE
C          POINT #1 TO BASE POINT #2, F10.3.
C                   COLS. 11-20; "XB1", HTM PLANE X COORDINATE OF BASE
C          POINT #1, F10.3.
C                   COLS. 21-30; "YB1", HTM PLANE Y COORDINATE OF BASE
C          POINT #1, F10.3.
C                   COLS. 31-40; "XB2", HTM PLANE X COORDINATE OF BASE
C          POINT #2, F10.3.
C                   COLS. 41-50; "YB2", HTM PLANE Y COORDINATE OF BASE
C          POINT #2, F10.3.
C          CARD #5: COLS. 1-10; "DB1P", HTM PLANE DISTANCE FROM BASE
C          POINT #1 TO UNKNOWN POINT, F10.3.
C                   COLS. 11-20; "DB2P", HTM PLANE DISTANCE FROM BASE
C          POINT #2 TO UNKNOWN POINT, F10.3.
C          CARD #6: COLS. 1-3; "ICMDEG", LONGITUDE (DEG.) OF CENTRAL
C          MERIDIAN OF ZONE, I3.
C                   COLS. 5-6; "ICMMIN", LONGITUDE (MIN.) OF CENTRAL
C          MERIDIAN OF ZONE, I2.
C                   COLS. 8-9; "ICMSEC", LONGITUDE (SEC.) OF CENTRAL
C          MERIDIAN OF ZONE, I2.
C                   COLS. 11-20; "OFFSET", X COORDINATE IN METERS OF
C          CENTRAL MERIDIAN OF ZONE, F10.3.
C          CARD #7: COLS. 1- 8, "H(1)", ELEVATION IN METERS OF BASE
C          POINT #1, F8.3.
C                   COLS. 9-16, "H(2)", ELEVATION IN METERS OF BASE
C          POINT #2, F8.3.
C                   COLS. 17-24, "H(3)", ELEVATION IN METERS OF UNKNOWN
C          POINT, F8.3.
C          OUTPUT CONSISTS OF THE FOLLOWING:
C          1. THE HEADER
C          2. THE BASELINE USED IN THE COMPUTATIONS
C          3. THE HTM PLANE DISTANCES INPUT TO THE PROGRAM
C          4. THE X & Y PLANE COORDINATES OF EACH POINT
C          5. THE GEOGRAPHICAL COORDINATES OF EACH POINT
C          6. THE CURVATURE RADIUS OF THE PRIME VERTICAL
C          7. THE USR COORDINATES OF EACH POINT
C          8. THE SPATIAL CHORD DISTANCES BETWEEN EACH PAIR OF POINTS
C          CALCULATED FROM THE USR COORDINATES OF EACH POINT
C
C

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CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C*** THIS SECTION INITIALIZES VARIABLES AND READS INPUT.
IMPLICIT REAL*6 (A-H,O-Z)
DIMENSION XA(20),POINT(4),BASE1(4),BASE2(4),H(3)
DATA XDIFF/10.0D-31/,A/6378206.4D0/,E2/0.006768658D0/
DATA IYPOS/'+'/,IYNEG/'-'/
PI=3.1415926535898D00
E4=E2**2
E6=E2**6
READ (8,100) XA
100 FORMAT (20A4)
WRITE (9,100) XA
READ (8,111) BASE1,BASE2,POINT
111 FORMAT (4A4,4A4,4A4)
READ (8,106) NTYPE,ISIGNY
106 FORMAT (T7,I1,T16,A1)
READ (8,107) DBL,XB1,YB1,XB2,YB2
107 FORMAT (5F10.3)
READ (8,108) DB1P,DB2P
108 FORMAT (2F10.3)
READ (8,101) ICMDEG,ICMMIN,ICMSEC,OFFSET
101 FORMAT (I3,1X,I2,1X,I2,1X,F10.3)
READ (8,113) H(1),H(2),H(3)
113 FORMAT (3F8.3)
WRITE (9,119) BASE1,BASE2
119 FORMAT (' BASE LINE FOR COMPUTATIONS IS',4A4,'TO',4A4)
WRITE (9,130) BASE1,BASE2,DBL,BASE1,POINT,DB1P,BASE2,POINT,DB2P
130 FORMAT (/, ' HTM PLANE DISTANCES (MD:',4A4,'TO',4A4,'=',F10.3,/,4
1A4,'TO',4A4,'=',F10.3,/,4A4,'TO',4A4,'=',F10.3)
C*** THIS SECTION COMPUTES THE X & Y PLANE COORDINATES OF BASE POINT #2.
DXX=XB2-XB1
DYY=YB2-YB1
IF (NTYPE .EQ. 2) GO TO 109
AZBASE=DATAN(DXX/DYY)
IF (AZBASE .LT. 0.) AZBASE=AZBASE+PI
XB2=XB1+DBL*DSIN(AZBASE)
YB2=YB1+DBL*DCOS(AZBASE)
DXX=XB2-XB1
DYY=YB2-YB1
109 CONTINUE
WRITE (9,117)
117 FORMAT (/, ' COORDINATES ON HTM PLANE')
WRITE (9,118) BASE1,XB1,BASE1,YB1,BASE2,XB2,BASE2,YB2
118 FORMAT (' 'X' COORDINATE OF',4A4,'=',F12.3,/, ' 'Y' COORDINATE
10F',4A4,'=',F12.3,/, ' 'X' COORDINATE OF',4A4,'=',F12.3,/, ' 'Y'
2 COORDINATE OF',4A4,'=',F12.3)
C*** THIS SECTION COMPUTES THE PLANE COORDINATES OF THE UNKNOWN POINT.
XPP=(DB1P**2-DB2P**2+DBL**2)/(2.*DBL)
YPP=DSQRT(DB1P**2-XPP**2)
IF (ISIGNY .EQ. IYNEG) YPP=-YPP
X3=XB1+XPP*DXX/DBL-YPP*DYY/DBL
Y3=YB1+XPP*DYY/DBL+YPP*DXX/DBL
WRITE (9,110) POINT,X3,POINT,Y3
110 FORMAT (' 'X' COORDINATE OF',4A4,'=',F12.3,/, ' 'Y' COORDINATE
10F',4A4,'=',F12.3)
C*** THIS LOOP COMPUTES GEOGRAPHICAL AND USR COORDINATES FOR EACH POINT.
DO 116 J=1,3
IF (J .EQ. 1) XXX=XB1
IF (J .EQ. 1) YYY=YB1
IF (J .EQ. 2) XXX=XB2
IF (J .EQ. 2) YYY=YB2
IF (J .EQ. 3) XXX=X3
IF (J .EQ. 3) YYY=Y3
C*** THIS SECTION INITIALIZES VARIABLES USED IN THE ITERATION PROCESS.

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CNER=PI/180.*(ICMDEG+ICMMIN/60.+ICMSEC/3600.)
DX=KXX-OFFSET
CL=(YYY/0.99996666667D0+2249134.918D0)/A
C1=1.D0-0.25D0*E2-3.D0/64.D0*E4-5.D0/256.D0*E6
C2=3.D0/8.D0*E2+3.D0/32.D0*E4+45.D0/1024D0*E6
C3=15.D0/256.D0*E4+45.D0/1024.D0*E6
C4=35.D0/3072.D0*E6
PHI=CL/C1
I=0
C*** THIS SECTION IS THE ITERATION LOOP TO FIND PHI', THE FOOTPOINT LATITUDE.
102 CONTINUE
I=I+1
CORR=-C2*DSIN(2.*PHI)+C3*DSIN(4.*PHI)-C4*DSIN(6.*PHI)
CL1=CL-CORR
RHI=CL1/C1
DIFF=DABS(RHI-PHI)
PHI=RHI
IF (DIFF .LE. XDIFF) GO TO 103
GO TO 102
103 CONTINUE
C*** THIS SECTION COMPUTES THE GEOGRAPHICAL COORDINATES, PHI & LAMDA.
W=DSQRT(1.D0-E2*DSIN(PHI)**2)
AM=A*(1.-E2)/W**3
AN=A/W
DLANDA=DX/(AN*DCOS(PHI)*0.99996666667D0)-(1.D0/(6.*AN**3*DCOS(PHI)
1))*(1.+2.*DTAN(PHI)**2)*(DX/0.99996666667D0)**3
PHI=PHI-DTAN(PHI)/(2.*AM*AN)*(DX/0.99996666667D0)**2+DTAN(PHI)/(24
1.*AM*AN**3)*(5.+3.*DTAN(PHI)**2)*(DX/0.99996666667D0)**4
DEC=PHI*180./PI
IDEC=IDINT(DEC)
AMIN=(DEC-IDEC)*60.
IMIN=IDINT(AMIN)
SEC=(AMIN-IMIN)*60.
ALANDA=CNER-DLANDA
DEC=ALANDA*180./PI
IDEC1=IDINT(DEC)
AMIN=(DEC-IDEC1)*60.
IMIN1=IDINT(AMIN)
SEC1=(AMIN-IMIN1)*60.
IF (J .EQ. 1) WRITE (9,112) BASE1
IF (J .EQ. 2) WRITE (9,112) BASE2
IF (J .EQ. 3) WRITE (9,112) POINT
112 FORMAT (' GEOGRAPHICAL COORDINATES OF',4A4)
IF (J .EQ. 3) WRITE (9,104) I
104 FORMAT (' NUMBER OF ITERATIONS TO FIND PHI':',I8)
WRITE (9,105) PHI, IDEC, IMIN, SEC, ALANDA, IDEC1, IMIN1, SEC1
105 FORMAT (' PHI (RADIANS)=' ,D17.11,3X, 'PHI (DEG.,MIN.,SEC.)=' ,I4
1,1X,I2,1X,F8.5,/, ' LAMDA (RADIANS)=' ,D17.11,3X, 'LAMDA (DEG.,MIN.,S
2EC.)=' ,I4,1X,I2,1X,F8.5)
C*** THIS SECTION COMPUTES THE USR COORDINATES.
W=DSQRT(1.D0-E2*DSIN(PHI)**2)
AN=A/W
X=(AN+H(J))*DCOS(PHI)*DCOS(ALANDA)
Y=-(AN+H(J))*DCOS(PHI)*DSIN(ALANDA)
Z=(AN*(1.-E2)+H(J))*DSIN(PHI)
IF (J .EQ. 1) AN1=AN
IF (J .EQ. 2) AN2=AN
IF (J .EQ. 1) XXB1=X
IF (J .EQ. 1) YYB1=Y
IF (J .EQ. 1) ZZB1=Z
IF (J .EQ. 2) XXB2=X
IF (J .EQ. 2) YYB2=Y
IF (J .EQ. 2) ZZB2=Z
116 CONTINUE

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WRITE (9,114) BASE1,AN1,XXB1,YYB1,ZZB1
WRITE (9,114) BASE2,AN2,XXB2,YYB2,ZZB2
WRITE (9,114) POINT,AN,X,Y,Z
114 FORMAT (/, ' UNIVERSAL SPACE RECTANGULAR COORDINATES OF',4A4,/, ' CU
IRVATURE RADIUS OF PRIME VERTICAL; N=',F13.3,/, ' X=',F13.3,/, ' Y=',
2F13.3,/, ' Z=',F13.3)
C*** THIS SECTION COMPUTES THE SPATIAL CHORD DISTANCE BETWEEN EACH PAIR
C*** OF POINTS FROM THE USR COORDINATES OF EACH POINT.
DDXX=XXB1-XXB2
DDYY=YYB1-YYB2
DDZZ=ZZB1-ZZB2
DDXX1=X-XXB1
DDYY1=Y-YYB1
DDZZ1=Z-ZZB1
DDXX2=X-XXB2
DDYY2=Y-YYB2
DDZZ2=Z-ZZB2
DIBL=DSQRT(DDXX**2+DDYY**2+DDZZ**2)
DIB1P=DSQRT(DDXX1**2+DDYY1**2+DDZZ1**2)
DIB2P=DSQRT(DDXX2**2+DDYY2**2+DDZZ2**2)
WRITE (9,115) BASE1,BASE2,DIBL,BASE1,POINT,DIB1P,BASE2,POINT,DIB2P
115 FORMAT (/, ' SPATIAL CHORD DISTANCE OF',4A4, 'TO',4A4, '=' ,F10.3,/, '
1SPATIAL CHORD DISTANCE OF',4A4, 'TO',4A4, '=' ,F10.3,/, ' SPATIAL CHOR
2D DISTANCE OF',4A4, 'TO',4A4, '=' ,F10.3)
999 STOP
END

```

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C          **** SPATIAL ****
C
C          PROGRAM TO MAKE A SPATIAL INTERSECTION AND COMPUTE THE LAT-
C          TITUDE, LONGITUDE, AND ELEVATION OF THE POINT IN QUESTION.
C          WRITTEN BY BRUCE SCHENCK, MARCH 1978.
C
C          PROGRAM TAKES THE UNIVERSAL SPACE RECTANGULAR COORDINATES OF
C          KNOWN POINTS, ALONG WITH THE APPROXIMATE COORDINATES OF THE
C          POINT IN QUESTION, AND USES AN ITERATIVE PROCESS TO SOLVE FOR
C          THE USR COORDINATES OF THE UNKNOWN POINT. THESE COORDINATES,
C          ALONG WITH AN APPROXIMATE LATITUDE AND CURVATURE RADIUS IN THE
C          PRIME VERTICAL, ARE THEN USED TO COMPUTE THE LATITUDE, LONGI-
C          TITUDE, AND ELEVATION OF THE POINT.
C          INPUT CARD FORMATS ARE AS FOLLOWS:
C          CARD #1: HEADER CARD WHICH WILL BE PRINTED AT TOP OF OUTPUT.
C          CARD #2: COLS. 3-4; "N", NUMBER OF KNOWN POINTS INPUT, I2.
C          CARD #3-#N+2: COLS. 1-10; DISTANCE IN KM FROM KNOWN POINT TO
C          UNKNOWN POINT, F10.6. COLS. 11-40; X, Y, Z COORDINATES OF KNOWN
C          POINT, 3F13.6.
C          CARD #N+3: COLS. 11-40; APPROXIMATE X, Y, Z COORDINATES OF
C          UNKNOW POINT, 3F13.6.
C          CARD #N+4: COLS. 1-12; "RN", RADIUS IN PRIME VERTICAL, F12.6.
C          COLS. 14-15; APPROXIMATE LATITUDE (DEG.) OF UNKNOW POINT, I2.
C          COLS. 17-18; APPROXIMATE LATITUDE (MIN.) OF UNKNOW POINT, I2.
C          COLS. 20-26; APPROXIMATE LATITUDE (SEC.) OF UNKNOW POINT, F7.4.
C          OUTPUT CONSISTS OF HEADER ON CARD #1, X, Y, Z COORDINATES
C          CALCULATED BY NEXT TO LAST ITERATION, X, Y, Z COORDINATES CALCU-
C          LATED BY LAST ITERATION, NUMBER OF ITERATIONS PERFORMED, LATI-
C          TITUDE, LONGITUDE, AND ELEVATIONS OF POINT CALCULATED FROM EACH OF
C          THE THREE COORDINATES.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C*** THIS SECTION INITIALIZES VARIABLES AND READS INPUT.
      IMPLICIT REAL*6 (A-H,O-Z)
      DIMENSION D(5),X(5),Y(5),Z(5),XX(20),Q(5),W(5),CC(5,5)
      NN=0
      MI=0
      PI=3.1415926535898D00
      READ (8,101) KX
101  FORMAT (20A4)
      READ (8,102) N
102  FORMAT (2X,I2)
      READ (8,100) (D(I),X(I),Y(I),Z(I),I=1,N)
100  FORMAT (F10.6,3F13.6)
      WRITE (9,101) XX
      READ (8,108) XBAR,YBAR,ZBAR
108  FORMAT (T11,3F13.6)
C*** THIS SECTION SETS UP X, Y, Z MATRIX.
103  CONTINUE
      DO 104 I=1,N
104  W(I)=((X(I)-XBAR)*X(I)+(Y(I)-YBAR)*Y(I)+(Z(I)-ZBAR)*Z(I))-D(I)**2
      DO 105 I=1,N
105  CC(I,1)=(X(I)-XBAR)
      DO 106 I=1,N
106  CC(I,2)=(Y(I)-YBAR)
      DO 107 I=1,N
107  CC(I,3)=(Z(I)-ZBAR)
C*** THIS SECTION CONSISTS OF A GAUSSIAN ELIMINATION PROCESS TO SOLVE
C*** THE MATRIX FOR X, Y, Z.
      IF(N.NE.1) GO TO 4
      IF(CC(1,1).EQ.0.) GO TO 3
      Q(1)=W(1)/CC(1,1)

```

```

GO TO 202
3 GO TO 203
4 NLESS1=N-1
DO 13 I=1,NLESS1
  BIC=ABS(CC(I,I))
  L=I.
  IPLUS1=I+1
  DO 6 J=IPLUS1,N
    IF (ABS(CC(J,I)) .LE. BIG) GO TO 6
    BIG=ABS(CC(J,I))
    L=J
6 CONTINUE
  IF (BIG .NE. 0.) GO TO 8
  GO TO 203
8 IF (L .EQ. I) GO TO 11
  DO 10 J=1,N
    TEMP=CC(L,J)
    CC(L,J)=CC(I,J)
10 CC(I,J)=TEMP
  TEMP=W(L)
  W(L)=W(I)
  W(I)=TEMP
11 DO 13 J=IPLUS1,N
  QUOT=CC(J,I)/CC(I,I)
  DO 12 M=IPLUS1,N
12 CC(J,M)=CC(J,M)-QUOT*CC(I,M)
13 W(J)=W(J)-QUOT*W(I)
  IF (CC(N,N) .NE. 0.) GO TO 15
  GO TO 203
15 Q(N)=W(N)/CC(N,N)
  I=N-1
16 SUM=0.
  IPLUS1=I+1
  DO 17 J=IPLUS1,N
17 SUM=SUM+CC(I,J)*Q(J)
  Q(I)=(W(I)-SUM)/CC(I,I)
  I=I-1
  IF (I .GT. 0) GO TO 16
202 CONTINUE
201 FORMAT (/, ' COORDINATES:',/, ' X=',F15.7,/, ' Y=',F15.7,/, ' Z=',F15.
17,/)
GO TO 205
203 WRITE (9,204)
204 FORMAT (' ***ERROR***=> IN CORRELATE MATRIX')
GO TO 999
C*** THIS SECTION CHECKS IF THE UNKNOWN X, Y, Z HAVE CONVERGED. IF NOT
C*** DO ITERATION AGAIN, OTHERWISE GO ON.
205 CONTINUE
  IF (NN .EQ. 1) GO TO 899
  IF (ABS(Q(1)-XBAR) .LT. 0.0000001 .AND. ABS(Q(2)-YBAR) .LT. 0.0000
1001 .AND. ABS(Q(3)-ZBAR) .LT. 0.0000001) NN=1
  IF (NN .GT. 1000) NN=1
  IF (NN .EQ. 1) WRITE (9,201) (Q(I),I=1,N)
  NN=NN+1
  XBAR=(Q(1)+XBAR)/2.
  YBAR=(Q(2)+YBAR)/2.
  ZBAR=(Q(3)+ZBAR)/2.
  GO TO 103
899 WRITE (9,201) (Q(I),I=1,N)
  WRITE (9,120) NN
120 FORMAT (' NUMBER OF ITERATIONS:',I5)
  READ (8,110) RN, IDEG, IMIN, SEC
110 FORMAT (F12.6, 1X, I2, 1X, I2, 1X, F7.4)
C*** THIS SECTION COMPUTES PHI, LAMDA, AND THE ELEVATIONS FROM EACH OF

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```

C** THE COORDINATES.
PHI=(SEC/3600.+IMIN/60.+IDEG)*PI/180.
ALAMDA=DATAN2(Q(1),Q(2))
HBAR=Q(1)/(DCOS(PHI)*DSIN(ALAMDA))-RN
PHI=DATAN(((6378.2064D0+HBAR)/(6356.5838D0+HBAR)**2*Q(3)*DSIN(ALA
1MDA)/Q(1))
HX=(Q(1)/(DCOS(PHI)*DSIN(ALAMDA)))-RN
HY=(Q(2)/(DCOS(PHI)*DCOS(ALAMDA)))-RN
HZ=(Q(3)/DSIN(PHI))-(RN*(6356.5838D0**2/6378.2064D0**2))
IF (ALAMDA.LT. 0.0) ALAMDA=PI*1.5+ALAMDA
PHI=PHI*180./PI
IDEG=INT(PHI)
AMIN=(PHI-IDEG)*60.
IMIN=INT(AMIN)
SEC=(AMIN-IMIN)*60.
ALAMDA=ALAMDA*180./PI
IDEG1=INT(ALAMDA)
AMIN=(ALAMDA-IDEG1)*60.
IMIN1=INT(AMIN)
SEC1=(AMIN-IMIN1)*60.
WRITE (9,111) IDEG,IMIN,SEC, IDEG1,IMIN1,SEC1,HX,HY,HZ
111 FORMAT (' PHI=',1X,I2,1X,I2,1X,F6.3,5X,'LAMDA=',1X,I3,1X,I2,1X,F6.
13,/, ' COMPUTED ELEVATIONS:',/, ' H(X)=' ,F10.6, ' KM  H(Y)=' ,F10.6, '
2 KM  H(Z)=' ,F10.6, ' KM')
999 STOP
END

```



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CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C          **** GRAVNET ****
C          PROGRAM TO ADJUST GRAVITY NETWORK BY A LEAST SQUARES METHOD;
C          WRITTEN BY BRUCE SCHENCK, JAN. 1978.
C
C          THIS PROGRAM TAKES AS INPUT MEASURED GRAVITY DIFFERENCES IN
C          MILLIGALS BETWEEN POINTS OF A NETWORK. FROM THE NETWORK LOOP
C          CONDITIONS ARE FORMED, AND THE MEASURED DIFFERENCES ARE THEN
C          ADJUSTED BY LEAST SQUARES SO THAT EACH LOOP OF THE NETWORK HAS
C          NO CLOSURE ERROR. AS THE PROGRAM IS WRITTEN NOW 80 MEASURED
C          DIFFERENCES CAN BE INPUT, AND 50 CONDITIONS CAN BE SOLVED. HOW-
C          EVER WITH A FEW CHANGES THESE VARIABLES CAN BE INCREASED. FOR
C          EACH NETWORK TO BE ADJUSTED THE CONDITIONS IN THE PROGRAM MUST
C          BE CHANGED ACCORDINGLY, ALONG WITH THE INITIALIZATION OF THE
C          CORRELATE MATRIX.
C          INPUT TO THE PROGRAM IS AS FOLLOWS:
C          CARD #1: HEADER CARD, ANYTHING ON THIS CARD WILL BE PRINTED AT
C                   THE TOP OF THE OUTPUT.
C          CARD #2: "NO", THE NUMBER OF MEASURED DIFFERENCE TO BE READ IN
C                   BY THE PROGRAM. IN AN I2 FORMAT STARTING IN COLUMN #4.
C          CARD #3: "N", THE NUMBER OF CONDITIONS PRESENT IN THE ADJUSTMENT
C                   IN AN I2 FORMAT STARTING IN COLUMN #3.
C          CARD #4-ON: COLS. 1-12; TERMINAL POINTS OF MEASURED DIFFERENCES
C                   IN A 2A6 FORMAT;
C                   COLS. 15-21; MEASURED GRAVITY DIFFERENCE IN MILLI-
C                   GALS, IN AN F7.0 FORMAT;
C                   COLS. 25-28; WEIGHT OF MEASUREMENT IN AN F4.0 FORMAT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C*** THIS SECTION INITIALIZES VARIABLES AND READS INPUT.
      DIMENSION ALINE(80),EX(12),XLINE(160),W(50),WT(80),R(80,50),
      1C(50,50),S(50),Q(50),WW(50),V(80)
      NN=0
      READ (8,100) EX
100  FORMAT (12A6)
      READ (8,101) NO
101  FORMAT (3X,12)
      READ (8,126) N
126  FORMAT (2X,12)
      READ (8,102) (XLINE(2*I-1),XLINE(2*I),ALINE(I),WT(I),I=1,NO)
102  FORMAT (2A6,T15,F7.0,T25,F4.0)
      WRITE (9,100) EX
      WRITE (9,121)
121  FORMAT (/, 'OBSERVED GRAVITY DIFFERENCES:')
119  CONTINUE
      WRITE (9,103)
      WRITE (9,104) (1,XLINE(2*I-1),XLINE(2*I),ALINE(I),WT(I),I=1,NO)
      IF (NN .EQ. 1) GO TO 122
103  FORMAT (' #',T6,'LINE:',T20,'GRAV. DIFF.:',T35,'WEIGHT:')
104  FORMAT (' ',I2,T6,2A6,T20,F7.2,T35,F4.1)
      DO 105 I=1,50
105  W(I)=0.
123  CONTINUE
C*** THIS SECTION CONTAINS THE CONDITIONS FOR THE ADJUSTMENT.
C====> NOTE: THE CONDITIONS MUST BE CHANGED FOR EACH DIFFERENT NETWORK.
C*** OAHU CONDITIONS PRESENT!
      W(1)=-ALINE(25)+ALINE(33)+ALINE(34)
      W(2)=-ALINE(25)-ALINE(32)+ALINE(36)
      W(3)=-ALINE(25)-ALINE(15)+ALINE(39)
      W(4)=-ALINE(25)+ALINE(24)-ALINE(26)
      W(5)=-ALINE(25)-ALINE(4)+ALINE(1)
      W(6)=-ALINE(25)-ALINE(4)+ALINE(2)

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W(7)=-ALINE(25)-ALINE(4)+ALINE(3)
W(8)=-ALINE(25)+ALINE(30)-ALINE(27)
W(9)=-ALINE(25)+ALINE(30)-ALINE(28)
W(10)=-ALINE(25)+ALINE(30)-ALINE(29)
W(11)=-ALINE(25)-ALINE(20)+ALINE(21)
W(12)=+ALINE(26)-ALINE(13)+ALINE(39)
W(13)=+ALINE(26)-ALINE(14)+ALINE(39)
W(14)=+ALINE(26)-ALINE(16)+ALINE(36)
W(15)=+ALINE(26)-ALINE(5)+ALINE(1)
W(16)=+ALINE(26)-ALINE(6)+ALINE(2)
W(17)=+ALINE(26)-ALINE(35)-ALINE(27)
W(18)=-ALINE(39)+ALINE(11)+ALINE(36)
W(19)=-ALINE(39)+ALINE(12)+ALINE(36)
W(20)=-ALINE(39)-ALINE(38)-ALINE(28)
W(21)=-ALINE(39)-ALINE(7)+ALINE(3)
W(22)=-ALINE(1)+ALINE(3)+ALINE(36)
W(23)=-ALINE(2)+ALINE(9)+ALINE(34)
W(24)=-ALINE(3)+ALINE(10)+ALINE(21)
W(25)=-ALINE(1)-ALINE(40)-ALINE(29)
W(26)=-ALINE(36)+ALINE(17)+ALINE(21)
W(27)=-ALINE(36)+ALINE(18)+ALINE(34)
W(28)=-ALINE(36)+ALINE(19)+ALINE(34)
W(29)=-ALINE(36)+ALINE(31)-ALINE(28)
W(30)=-ALINE(34)-ALINE(23)+ALINE(21)
W(31)=-ALINE(34)+ALINE(37)-ALINE(29)
W(32)=-ALINE(21)+ALINE(22)-ALINE(27)
C*** MAUI CONDITIONS PRESENT!
      IF (NN.EQ. 1) GO TO 124
C*** THIS SECTION COMPUTES THE CONDITION RESIDUALS.
      WRITE (9,106)
106  FORMAT (/,' CONDITION RESIDUALS BEFORE ADJUSTMENT:')
      WRITE (9,107) (I,W(I),I=1,N)
107  FORMAT (13,' ',F5.2)
      DO 150 I=1,N
150  WW(I)=W(I)
C*** THIS SECTION INITIALIZES THE CORRELATE MATRIX.
      DO 108 I=1,50
      DO 108 J=1,80
108  R(J,I)=0.
C====> NOTE: THE INITIALIZATION OF THE CORRELATE MATRIX MUST BE CHANGED
C          FOR EACH NETWORK.
C*** CONDITION #1; INITIALIZE
      R(25,1)=-1.
      R(33,1)=+1.
      R(34,1)=+1.
C*** CONDITION #2; INITIALIZE
      R(25,2)=-1.
      R(32,2)=-1.
      R(36,2)=+1.
C*** CONDITION #3; INITIALIZE
      R(25,3)=-1.
      R(15,3)=-1.
      R(39,3)=+1.
C*** CONDITION #4; INITIALIZE
      R(25,4)=-1.
      R(24,4)=+1.
      R(26,4)=-1.
C*** CONDITION #5; INITIALIZE
      R(25,5)=-1.
      R(4,5)=-1.
      R(1,5)=+1.
C*** CONDITION #6; INITIALIZE
      R(25,6)=-1.
      R(4,6)=-1.

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R(2,6)=+1.
C*** CONDITION #7; INITIALIZE
R(25,7)=-1.
R(4,7)=-1.
R(3,7)=+1.
C*** CONDITION #8; INITIALIZE
R(25,8)=-1.
R(30,8)=+1.
R(27,8)=-1.
C*** CONDITION #9; INITIALIZE
R(25,9)=-1.
R(30,9)=+1.
R(28,9)=-1.
C*** CONDITION #10; INITIALIZE
R(25,10)=-1.
R(30,10)=+1.
R(29,10)=-1.
C*** CONDITION #11; INITIALIZE
R(25,11)=-1.
R(20,11)=-1.
R(21,11)=+1.
C*** CONDITION #12; INITIALIZE
R(26,12)=+1.
R(13,12)=-1.
R(39,12)=+1.
C*** CONDITION #13; INITIALIZE
R(26,13)=+1.
R(14,13)=-1.
R(39,13)=+1.
C*** CONDITION #14; INITIALIZE
R(26,14)=+1.
R(16,14)=-1.
R(36,14)=+1.
C*** CONDITION #15; INITIALIZE
R(26,15)=+1.
R(5,15)=-1.
R(1,15)=+1.
C*** CONDITION #16; INITIALIZE
R(26,16)=+1.
R(6,16)=-1.
R(2,16)=+1.
C*** CONDITION #17; INITIALIZE
R(26,17)=+1.
R(35,17)=-1.
R(27,17)=-1.
C*** CONDITION #18; INITIALIZE
R(39,18)=-1.
R(11,18)=+1.
R(36,18)=+1.
C*** CONDITION #19; INITIALIZE
R(39,19)=-1.
R(12,19)=+1.
R(36,19)=+1.
C*** CONDITION #20; INITIALIZE
R(39,20)=-1.
R(38,20)=-1.
R(28,20)=-1.
C*** CONDITION #21; INITIALIZE
R(39,21)=-1.
R(7,21)=-1.
R(3,21)=+1.
C*** CONDITION #22; INITIALIZE
R(1,22)=-1.
R(8,22)=+1.
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R(34,22)=+1.
C*** CONDITION #23; INITIALIZE
R(2,23)=-1.
R(9,23)=+1.
R(34,23)=+1.
C*** CONDITION #24; INITIALIZE
R(3,24)=-1.
R(10,24)=+1.
R(21,24)=+1.
C*** CONDITION #25; INITIALIZE
R(1,25)=-1.
R(40,25)=-1.
R(29,25)=-1.
C*** CONDITION #26; INITIALIZE
R(36,26)=-1.
R(17,26)=+1.
R(21,26)=+1.
C*** CONDITION #27; INITIALIZE
R(36,27)=-1.
R(18,27)=+1.
R(34,27)=+1.
C*** CONDITION #28; INITIALIZE
R(36,28)=-1.
R(19,28)=+1.
R(34,28)=+1.
C*** CONDITION #29; INITIALIZE
R(36,29)=-1.
R(31,29)=+1.
R(28,29)=-1.
C*** CONDITION #30; INITIALIZE
R(34,30)=-1.
R(23,30)=-1.
R(21,30)=+1.
C*** CONDITION #31; INITIALIZE
R(34,31)=-1.
R(37,31)=+1.
R(29,31)=-1.
C*** CONDITION #32; INITIALIZE
R(21,32)=-1.
R(22,32)=+1.
R(27,32)=-1.
C*** THIS SECTION SETS UP THE NORMAL EQUATIONS.
DO 109 I=1,50
DO 109 J=1,50
109 C(I,J)=0.
DO 110 K=1,N
DO 110 J=1,N
DO 110 I=1,NO
S(I)=R(I,K)*R(I,J)/WT(I)
110 C(K,J)=C(K,J)+S(I)
C*** THIS SECTION CONSISTS OF A GAUSSIAN ELIMINATION PROCESS TO SOLVE
C*** THE NORMAL EQUATIONS FOR THE CORRELATES.
IF (N .NE. 1) GO TO 4
IF (C(1,1) .EQ. 0.) GO TO 3
Q(1)=W(1)/C(1,1)*(-1.)
GO TO 202
3 GO TO 203
4 NLESS1=N-1
DO 13 I=1,NLESS1
BIG=ABS(C(I,I))
L=I
IPLUS1=I+1
DO 6 J=IPLUS1,N
IF (ABS(C(J,I)) .LE. BIG) GO TO 6

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```

BIG=ABS(C(J,I))
L=J
6 CONTINUE
IF (BIG .NE. 0.) GO TO 8
GO TO 203
8 IF (L .EQ. I) GO TO 11
DO 10 J=1,N
TEMP=C(L,J)
C(L,J)=C(I,J)
10 C(I,J)=TEMP
TEMP=W(L)
W(L)=W(I)
W(I)=TEMP
11 DO 13 J=IPLUS1,N
QUOT=C(J,I)/C(I,I)
DO 12 K=IPLUS1,N
12 C(J,K)=C(J,K)-QUOT*C(I,K)
13 W(J)=W(J)-QUOT*W(I)
IF (C(N,N) .NE. 0.) GO TO 15
GO TO 203
15 Q(N)=W(N)/C(N,N)
I=N-1
16 SUM=0.
IPLUS1=I+1
DO 17 J=IPLUS1,N
17 SUM=SUM+C(I,J)*Q(J)
Q(I)=(W(I)-SUM)/C(I,I)
I=I-1
IF (I .GT. 0.) GO TO 16
DO 18 KK=1,N
18 Q(KI)=-Q(KI)
202 WRITE (9,201) (I,Q(I),I=1,N)
201 FORMAT (' CORRELATES:',/,25(I3,':',F15.3,/,),/)
GO TO 205
203 WRITE (9,204)
204 FORMAT (' ***ERROR***=> IN CORRELATE MATRIX')
GO TO 999
205 CONTINUE
SUMVV=0.
DO 111 I=1,N
111 SUMVV=SUMVV-Q(I)*WW(I)
C*** THIS SECTION COMPUTES THE CORRECTIONS TO EACH MEASUREMENT.
DO 112 I=1,80
112 V(I)=0.
DO 113 I=1,NO
DO 113 J=1,N
VV=R(I,J)*Q(J)
113 V(I)=V(I)+VV
DO 114 I=1,NO
114 V(I)=V(I)/WT(I)
WRITE (9,115) (I,V(I),I=1,NO)
115 FORMAT (' CORRECTIONS TO LINES:',/,50(I3,':',F7.3,/,))
C*** THIS SECTION COMPUTES SUM VV , A CHECK ON SUM VV, AND MU.
SUMVV2=0.
DO 116 I=1,NO
GI=V(I)**2
116 SUMVV2=SUMVV2+GI
AMU=SQRT(SUMVV2/N)
WRITE (9,117) SUMVV,SUMVV2,AMU
117 FORMAT (' SUM VV=',F10.3,/, ' CHECK ON SUM VV=',F10.3,/, ' MU=',F10.
13,/)
C*** THIS SECTION COMPUTES EACH ADJUSTED MEASUREMENT.
DO 118 I=1,NO
118 ALINE(I)=ALINE(I)+V(I)

```

```
      NN=NN+1
      WRITE (9,120)
120  FORMAT (' CORRECTED GRAVITY DIFFERENCES:')
      GO TO 119
122  CONTINUE
C*** THIS SECTION COMPUTES THE CONDITION RESIDUALS AFTER ADJUSTMENT.
      GO TO 123
124  CONTINUE
      WRITE (9,125)
125  FORMAT (/, ' CONDITION RESIDUALS AFTER ADJUSTMENT:')
      WRITE (9,107) (I,W(I),I=1,N)
999  STOP
      END
```

## MAUI GRAVITY NETWORK

## OBSERVED GRAVITY DIFFERENCES:

#	LINE:	GRAV. DIFF.:	WEIGHT:
1	#1 TO #3	27.45	1.0
2	#1 TO #3	27.44	1.0
3	#1 TO #3	27.45	1.0
4	#1 TO #3	27.46	1.0
5	#1 TO #3	27.45	1.0
6	#1 TO #3	27.44	1.0
7	#1 TO #3	27.44	1.0
8	#3 TO #5	68.52	1.0
9	#3 TO #5	68.53	1.0
10	#3 TO #5	68.53	1.0
11	#3 TO #5	68.54	1.0
12	#3 TO #5	68.52	1.0
13	#3 TO #5	68.53	1.0
14	#5 TO #15	321.92	1.0
15	#5 TO #15	321.89	1.0
16	#5 TO #15	321.89	1.0
17	#15 TO #21	240.66	1.0
18	#15 TO #21	240.65	1.0
19	#15 TO #21	240.68	1.0
20	#3 TO H.B.	-78.92	1.0
21	H.B. TO HAP	9.98	1.0
22	#3 TO HAP	-69.01	1.0
23	#3 TO HAP	-68.99	1.0
24	#3 TO L.P.	-37.45	1.0
25	#3 TO L.P.	-37.46	1.0
26	#1 TO L.P.	-9.98	1.0
27	#1 TO L.P.	-9.97	1.0
28	#5 TO L.P.	-105.98	1.0
29	#5 TO L.P.	-105.98	1.0
30	#15 TO L.P.	-427.87	1.0
31	#15 TO L.P.	-427.91	1.0
32	#2 TO #21	663.70	1.0
33	#3 TO #15	390.41	1.0
34	#2 TO #3	32.64	1.0
35	#2 TO L.P.	-4.84	1.0
36	#2 TO L.P.	-4.83	1.0
37	#21 TO L.P.	-668.58	1.0
38	#2 TO #5	101.15	1.0
39	#1 TO #2	-5.16	1.0
40	#3 TO #21	631.08	1.0
41	#3 TO #21	631.10	1.0
42	#21 TO H.B.	-710.00	1.0
43	#21 TO HAP	-700.09	1.0

## CONDITION RESIDUALS BEFORE ADJUSTMENT:

1: 0.02  
 2: 0.03  
 3: 0.01  
 4: 0.03  
 5: 0.01  
 6: 0.01  
 7: 0.00  
 8: -0.05  
 9: -0.02  
 10: -0.05  
 11: -0.02  
 12: -0.03  
 13: -0.02  
 14: -0.05

15: -0.03  
 16: -0.01  
 17: 0.01  
 18: 0.00  
 19: 0.02  
 20: -0.01  
 21: 0.03  
 22: -0.01  
 23: -0.04  
 24: -0.05  
 25: -0.02  
 26: -0.01  
 27: 0.00  
 28: -0.04  
 29: -0.05  
 30: -0.02  
 31: -0.03  
 32: -0.07  
 33: -0.05  
 34: 0.00  
 35: -0.02

CORRELATES:

1: 0.077  
 2: -0.005  
 3: -0.067  
 4: -0.043  
 5: 0.023  
 6: -0.036  
 7: 0.035  
 8: 0.013  
 9: 0.005  
 10: 0.015  
 11: 0.005  
 12: -0.005  
 13: 0.005  
 14: 0.015  
 15: 0.015  
 16: 0.010  
 17: 0.000  
 18: 0.000  
 19: -0.010  
 20: 0.010  
 21: -0.020  
 22: -0.034  
 23: 0.053  
 24: 0.003  
 25: 0.017

26: -0.021  
 27: 0.009  
 28: 0.009  
 29: 0.004  
 30: 0.014  
 31: -0.016  
 32: 0.033  
 33: 0.003  
 34: -0.010  
 35: 0.024

CORRECTIONS TO LINES:

1: 0.005  
 2: 0.015  
 3: 0.005



4: -0.005  
 5: 0.005  
 6: 0.015  
 7: 0.015  
 8: 0.010  
 9: 0.000  
 10: 0.000  
 11: -0.010  
 12: 0.010  
 13: -0.020  
 14: -0.021  
 15: 0.009  
 16: 0.009  
 17: 0.004  
 18: 0.014  
 19: -0.016  
 20: -0.011  
 21: -0.036  
 22: 0.023  
 23: 0.003  
 24: -0.003  
 25: 0.007  
 26: -0.019  
 27: -0.029  
 28: -0.003  
 29: -0.003  
 30: -0.013  
 31: 0.027  
 32: 0.013  
 33: 0.019  
 34: -0.020  
 35: 0.006  
 36: -0.004  
 37: 0.004  
 38: 0.000  
 39: -0.005  
 40: 0.013  
 41: -0.007  
 42: -0.024  
 43: 0.010

SUM VV= 0.010  
 CHECK ON SUM VV= 0.010  
 MU= 0.017

CORRECTED GRAVITY DIFFERENCES:			
#	LINE:	GRAV. DIFF.:	WEIGHT:
1	#1 TO #3	27.45	1.0
2	#1 TO #3	27.45	1.0
3	#1 TO #3	27.45	1.0
4	#1 TO #3	27.45	1.0
5	#1 TO #3	27.45	1.0
6	#1 TO #3	27.45	1.0
7	#1 TO #3	27.45	1.0
8	#3 TO #5	68.53	1.0
9	#3 TO #5	68.53	1.0
10	#3 TO #5	68.53	1.0
11	#3 TO #5	68.53	1.0
12	#3 TO #5	68.53	1.0
13	#3 TO #5	68.53	1.0
14	#5 TO #15	321.90	1.0
15	#5 TO #15	321.90	1.0
16	#5 TO #15	321.90	1.0
17	#15 TO #21	240.66	1.0

18	#15 TO #21	240.66	1.0
19	#15 TO #21	240.66	1.0
20	#3 TO H.B.	-78.93	1.0
21	H.B. TO HAP	9.94	1.0
22	#3 TO HAP	-68.99	1.0
23	#3 TO HAP	-68.99	1.0
24	#3 TO L.P.	-37.45	1.0
25	#3 TO L.P.	-37.45	1.0
26	#1 TO L.P.	-10.00	1.0
27	#1 TO L.P.	-10.00	1.0
28	#5 TO L.P.	-105.98	1.0
29	#5 TO L.P.	-105.98	1.0
30	#15 TO L.P.	-427.88	1.0
31	#15 TO L.P.	-427.88	1.0
32	#2 TO #21	663.71	1.0
33	#3 TO #15	390.43	1.0
34	#2 TO #3	32.62	1.0
35	#2 TO L.P.	-4.83	1.0
36	#2 TO L.P.	-4.83	1.0
37	#21 TO L.P.	-668.55	1.0
38	#2 TO #5	101.15	1.0
39	#1 TO #2	-5.17	1.0
40	#3 TO #21	631.09	1.0
41	#3 TO #21	631.09	1.0
42	#21 TO H.B.	-710.02	1.0
43	#21 TO HAP	-700.08	1.0

## CONDITION RESIDUALS AFTER ADJUSTMENT:

1: 0.00  
2: 0.00  
3: 0.00  
4: 0.00  
5: 0.00  
6: 0.00  
7: 0.00  
8: 0.00  
9: 0.00  
10: 0.00  
11: 0.00  
12: 0.00  
13: 0.00  
14: 0.00  
15: 0.00  
16: 0.00  
17: 0.00  
18: 0.00  
19: 0.00  
20: 0.00  
21: 0.00  
22: 0.00  
23: 0.00  
24: 0.00  
25: 0.00  
26: 0.00  
27: 0.00  
28: 0.00  
29: 0.00  
30: 0.00  
31: 0.00  
32: 0.00  
33: 0.00  
34: 0.00  
35: 0.00

## OAHU GRAVITY NETWORK

## OBSERVED GRAVITY DIFFERENCES:

#	LINE:	GRAV. DIFF.:	WEIGHT:
1	HIG TO I. I.	25.80	1.0
2	HIG TO I. I.	25.79	1.0
3	HIG TO I. I.	25.83	1.0
4	HIG TO B.M.	5.87	1.0
5	HIG TO W/M	5.22	1.0
6	HIG TO W/M	5.25	1.0
7	HIG TO #47	-7.89	1.0
8	HIG TO #171	3.48	1.0
9	HIG TO #324	20.04	1.0
10	HIG TO #325	27.66	1.0
11	#47 TO #171	11.36	1.0
12	#47 TO #171	11.40	1.0
13	#47 TO W/M	13.12	1.0
14	#47 TO W/M	13.12	1.0
15	#47 TO B.M.	13.74	1.0
16	#171 TO W/M	1.73	1.0
17	#171 TO #325	24.14	1.0
18	#171 TO #324	16.54	1.0
19	#171 TO #324	16.51	1.0
20	#325 TO B.M.	-21.74	1.0
21	#325 TO I. I.	-1.85	1.0
22	#325 TO HICK	-0.43	1.0
23	#325 TO #324	-7.60	1.0
24	B.M. TO W/M	-0.63	1.0
25	B.M. TO I. I.	19.92	1.0
26	I. I. TO W/M	-20.57	1.0
27	I. I. TO HICK	1.40	1.0
28	I. I. TO HICK	1.37	1.0
29	I. I. TO HICK	1.37	1.0
30	B.M. TO HICK	21.32	1.0
31	#171 TO HICK	23.75	1.0
32	#171 TO B.M.	2.38	1.0
33	B.M. TO #324	14.14	1.0
34	#324 TO I. I.	5.75	1.0
35	HICK TO W/M	-21.97	1.0
36	#171 TO I. I.	22.27	1.0
37	#324 TO HICK	7.12	1.0
38	HICK TO #47	-35.06	1.0
39	#47 TO I. I.	33.69	1.0
40	HICK TO HIG	-27.21	1.0

## CONDITION RESIDUALS BEFORE ADJUSTMENT:

1: -0.03  
 2: -0.03  
 3: 0.03  
 4: 0.02  
 5: 0.01  
 6: 0.00  
 7: 0.04  
 8: 0.00  
 9: 0.03  
 10: 0.03  
 11: -0.03  
 12: 0.00  
 13: 0.00  
 14: -0.03  
 15: 0.01  
 16: -0.03  
 17: 0.00

18: -0.06  
 19: -0.02  
 20: 0.00  
 21: 0.03  
 22: -0.05  
 23: 0.00  
 24: -0.02  
 25: 0.04  
 26: 0.02  
 27: 0.02  
 28: -0.01  
 29: 0.11  
 30: 0.00  
 31: 0.00  
 32: 0.02

## CORRELATES:

1: 0.016  
 2: 0.002  
 3: -0.017  
 4: -0.012  
 5: 0.038  
 6: -0.008  
 7: -0.039  
 8: 0.030  
 9: 0.017  
 10: -0.050  
 11: 0.020  
 12: 0.005  
 13: 0.005  
 14: -0.006  
 15: -0.016  
 16: 0.014  
 17: -0.011  
 18: 0.019  
 19: -0.021  
 20: 0.014  
 21: -0.011  
 22: 0.034  
 23: -0.006  
 24: -0.022  
 25: -0.015

26: -0.002  
 27: -0.006  
 28: 0.024  
 29: -0.055  
 30: 0.004  
 31: 0.041  
 32: -0.013

## CORRECTIONS TO LINES:

1: 0.002  
 2: 0.012  
 3: -0.028  
 4: 0.008  
 5: 0.016  
 6: -0.014  
 7: 0.011  
 8: 0.034  
 9: -0.006  
 10: -0.022  
 11: 0.019  
 12: -0.021

13: -0.005  
 14: -0.005  
 15: 0.017  
 16: 0.006  
 17: -0.002  
 18: -0.006  
 19: 0.024  
 20: -0.020  
 21: 0.014  
 22: -0.013  
 23: -0.004  
 24: -0.012  
 25: 0.004  
 26: 0.004  
 27: -0.007  
 28: 0.023  
 29: 0.023  
 30: -0.003  
 31: -0.055  
 32: -0.002  
 33: 0.016  
 34: 0.018  
 35: 0.011  
 36: 0.032  
 37: 0.041  
 38: -0.014  
 39: -0.009  
 40: 0.015

SUM VV= 0.014  
 CHECK ON SUM VV= 0.014  
 NU= 0.021

## CORRECTED GRAVITY DIFFERENCES:

#	LINE:	GRAV. DIFF.:	WEIGHT:
1	HIG TO I. I.	25.80	1.0
2	HIG TO I. I.	25.80	1.0
3	HIG TO I. I.	25.80	1.0
4	HIG TO B.M.	5.38	1.0
5	HIG TO W/M	5.24	1.0
6	HIG TO W/M	5.24	1.0
7	HIG TO #47	-7.88	1.0
8	HIG TO #171	3.51	1.0
9	HIG TO #324	20.93	1.0
10	HIG TO #325	27.64	1.0
11	#47 TO #171	11.38	1.0
12	#47 TO #171	11.38	1.0
13	#47 TO W/M	13.11	1.0
14	#47 TO W/M	13.11	1.0
15	#47 TO B.M.	13.76	1.0
16	#171 TO W/M	1.74	1.0
17	#171 TO #325	24.14	1.0
18	#171 TO #324	16.53	1.0
19	#171 TO #324	16.53	1.0
20	#325 TO B.M.	-21.76	1.0
21	#325 TO I. I.	-1.84	1.0
22	#325 TO HICK	-0.44	1.0
23	#325 TO #324	-7.60	1.0
24	B.M. TO W/M	-0.64	1.0
25	B.M. TO I. I.	19.92	1.0
26	I. I. TO W/M	-20.57	1.0
27	I. I. TO HICK	1.39	1.0
28	I. I. TO HICK	1.39	1.0
29	I. I. TO HICK	1.39	1.0

30	B.M. TO HICK	21.32	1.0
31	#171 TO HICK	23.70	1.0
32	#171 TO B.M.	2.38	1.0
33	B.M. TO #324	14.16	1.0
34	#324 TO I. I.	5.77	1.0
35	HICK TO W/M	-21.96	1.0
36	#171 TO I. I.	22.30	1.0
37	#324 TO HICK	7.16	1.0
38	HICK TO #47	-35.07	1.0
39	#47 TO I. I.	33.68	1.0
40	HICK TO HIG	-27.20	1.0

## CONDITION RESIDUALS AFTER ADJUSTMENT:

1: 0.00  
2: 0.00  
3: 0.00  
4: 0.00  
5: 0.00  
6: 0.00  
7: 0.00  
8: 0.00  
9: 0.00  
10: 0.00  
11: 0.00  
12: 0.00  
13: 0.00  
14: 0.00  
15: 0.00  
16: 0.00  
17: 0.00  
18: 0.00  
19: 0.00  
20: 0.00  
21: 0.00  
22: 0.01  
23: 0.00  
24: 0.00  
25: 0.00  
26: 0.00  
27: 0.00  
28: 0.00  
29: 0.00  
30: 0.00  
31: 0.00  
32: 0.00