THUNDERSTORMS IN HAWAII

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ABSTRACT

Thunderstorm occurrences are compared with synoptic patterns (proximity of fronts, cyclones, and flow patterns). Only very small numbers of thunderstorms are found to be associated with these large scale patterns.

The relationship of diabatic effects, convection, and horizontal advection to temperature changes is discussed and some computations are compared with observed data. It is found that the advective tendency is not related to the observed temperature changes. The observed changes probably result from compensating effects produced by non-adiabatic and convective influences which may be an order of magnitude larger than the advective contribution. The orders of magnitude of convective cooling and radiative cooling at cloud tops are estimated by calculations. Although the models are crude, they indicate that vertical motion is the most important destabilizing influence.

Some case studies of air-mass structure are made through the use of virtual-equivalent-potential temperature and relative humidity time sections. Soundings are found to be convectively unstable at all times, despite the presence of the trade inversion above the moist layer, but thunderstorms are more prevalent when the moist layer is deep and the trade inversion is weak or non-existent. Since a relatively small amount of lifting is necessary to release the convective instability the mountainous terrain would

produce frequent thunderstorms unless cumulus development
is strongly inhibited by other, perhaps related, condi-
tions.   Entrainment of dry air has the desired effect,
but numerical computations based on a model which simu-
lates cumulus dynamics including entrainment show that
entrainment is a necessary but not sufficient condition
to explain the relatively infrequent occurrences of
thunderstorms.

THUNDERSTORMS IN HAWAII

INTRODUCTION

The annual and semi-diurnal thunderstorm frequencies
(Figs. 1-3) in the Hawaiian Islands have been described by
Lee (1967). These frequencies revealed marked periodici-
ties suggesting more than casual relationships with both
synoptic and microscale phenomena. Thunderstorm occurrences
are highest during the cool months from October through
March. The frequency generally attains a maximum during
March. This seasonal variation suggests a correlation
with the more common wintertime disturbances, viz., fronts
and cyclones. It was shown also that the diurnal thunder-
storm frequency is bimodal in structure, having distinct
maxima during the mid-afternoon and early morning hours.
The earlier study suggested radiative cooling as a probable
factor in the production of the nocturnal maxima.

In the present study the suggested relationships between
thunderstorms and large scale synoptic features are first
examined statistically. When one or more thunderstorms or
lightning is reported at either Lihue, Honolulu, or Hilo
the day is classified as a "thunderstorm day". The number
of thunderstorm and non-thunderstorm days is then determined
when fronts, and cyclones are in proximity to these stations.
The numbers are also determined for typical flow patterns.
The inference of cold advection aloft as an initiator of
thunderstorms leads to a discussion of the relationship of
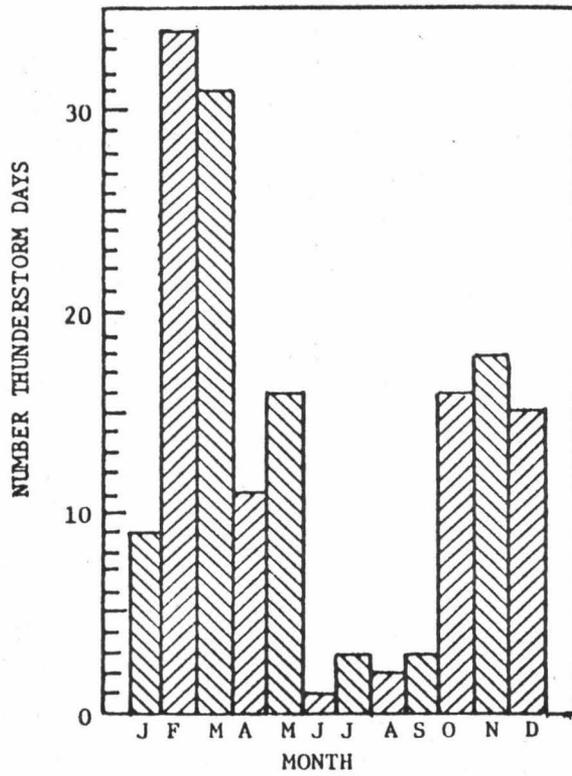
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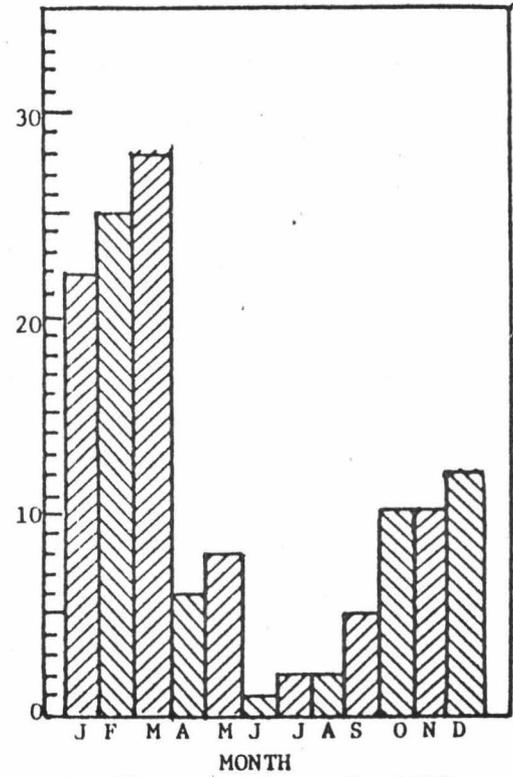
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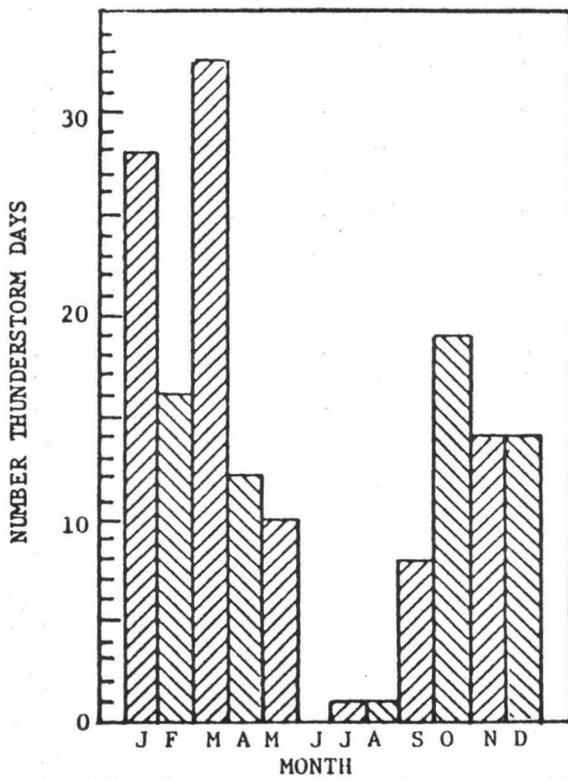
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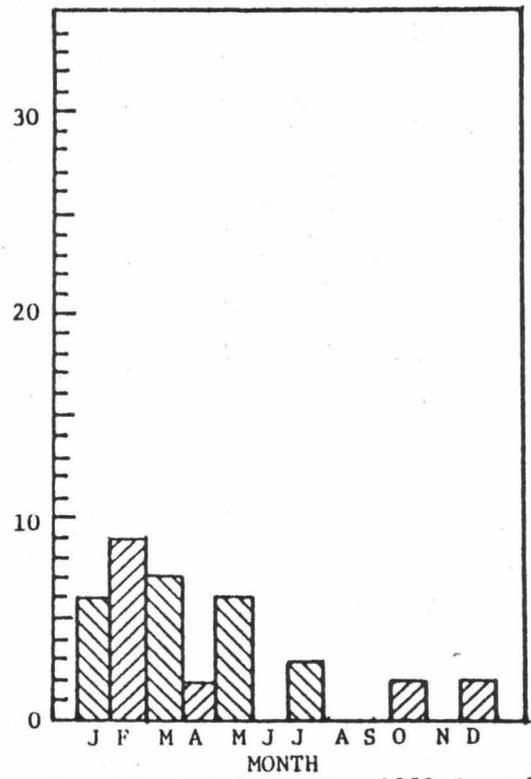
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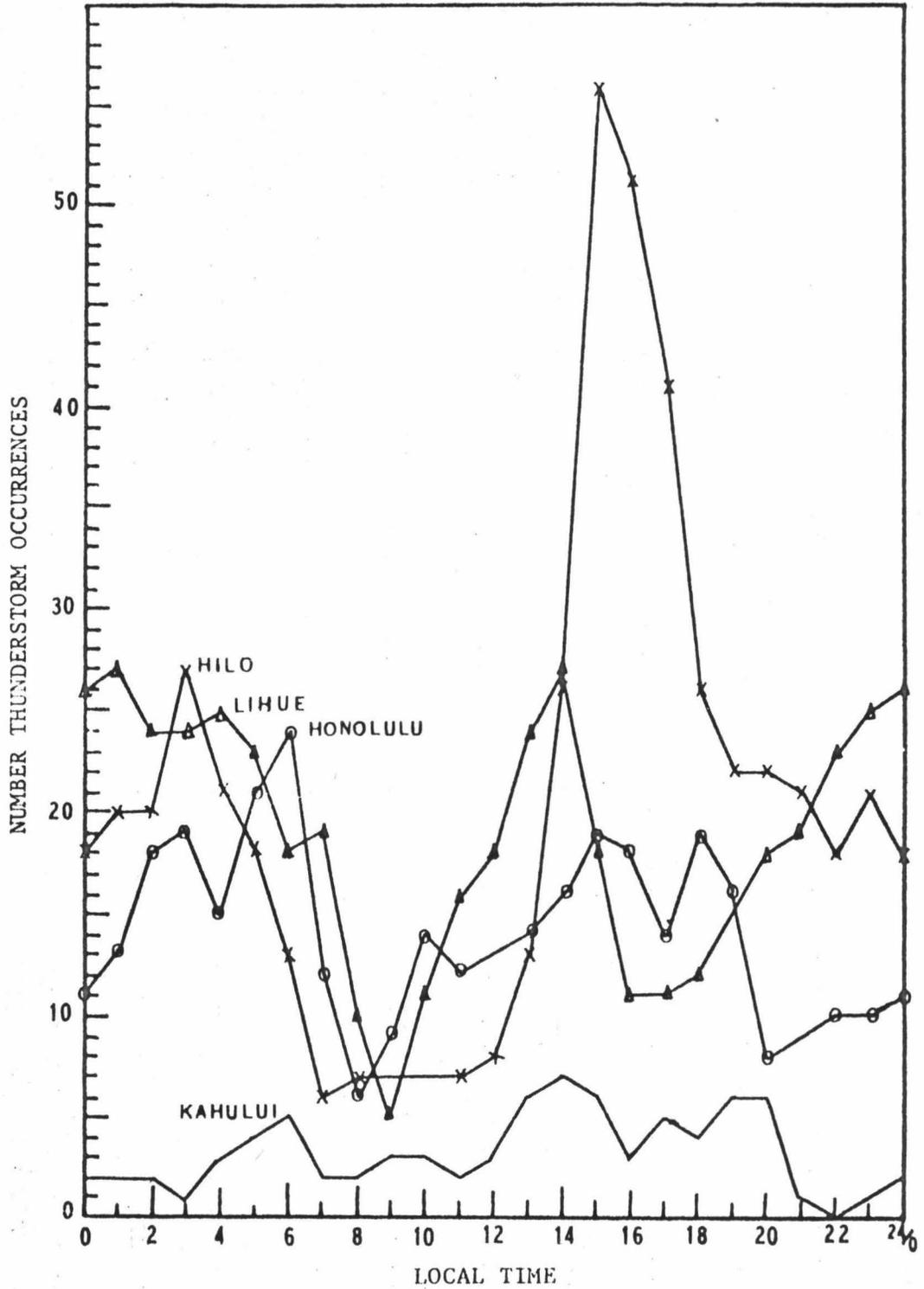
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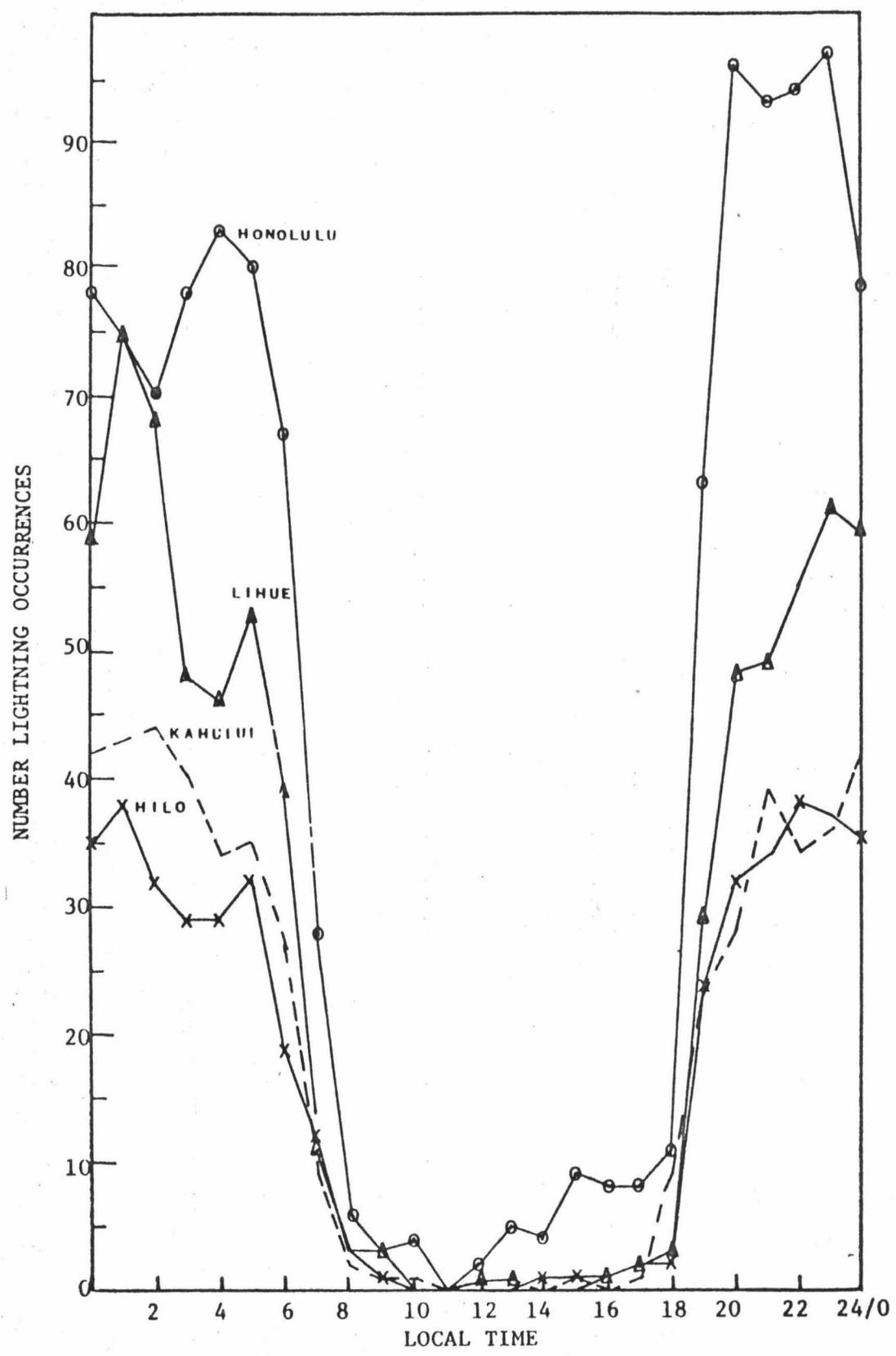
FIG. 3. DIURNAL LIGHTNING DISTRIBUTION

vertical motion, radiation, and horizontal advection to temperature changes. Estimates are made of the relative orders of magnitude of these three factors. Next, investigation of the vertical structure of the air mass, through the use of virtual-equivalent-potential temperature and relative humidity diagrams, reveals highly favorable conditions for thunderstorms but only infrequent occurrences. The relative rarity of thunderstorms suggests entrainment as a probable factor suppressing the production of more frequent thunderstorm activity. This suggestion is tested through the use of a numerical model which simulates cumulus dynamics and the effects of entrainment.

DISCUSSION OF METHODS AND RESULTS

FRONTAL DAYS

An analysis of frontal cloud bands from 20-30N and 155-180W taken from satellite nephanalyses during the period 1965-66 showed average frontal cloud widths of about 350 km. The rectangular area shown in Fig. 4 is chosen slightly larger to account for some variability and to include all of the Hawaiian chain. Frontal-days, i.e., when the mean position of the front is within the area (Worthley, 1967), are tabulated against the occurrence or non-occurrence of thunderstorm-days.

The results are shown in Table 1. Of a total of 618 frontal-days thunderstorms or lightning were reported on

Month	No. of frontal-days with a thunderstorm-day	No. of frontal-days without a thunderstorm-day	Total
Sep	3	10	13
Oct	9	40	49
Nov	6	57	66
Dec	18	89	107
Jan	22	126	148
Feb	7	80	87
Mar	22	53	75
Apr	5	53	58
May	5	11	16
Jun	0	2	2
Total	97	521	618

Table 1. Frequency of frontal-days with thunderstorm-days at Lihue, Honolulu or Hilo. Jan. 1950-Sept. 1966.

only 97 days or about 16 per cent of all cases. The ratio of thunderstorm-days to frontal-days during November is 1:11, December 1:6, January 1:7, February 1:12, March 1:3, and April 1:12. March, which has the highest ratio, is the month of peak thunderstorm frequency at Lihue and Honolulu. January contained nearly twice as many frontal-days as March, yet it experienced less than half the frequency of thunderstorm-days. A similar unexpected feature is observed in the low frequency of thunderstorm-days during February as compared to March although the number of frontal-days were similar.

CYCLONE DAYS

The remaining thunderstorm-days, when fronts were not present, are compared with days when cyclones were reported in the vicinity of the Hawaiian Islands (Worthley, 1967). The area chosen is larger than that for the fronts (see Fig. 4). When cyclones were analyzed in the area on one or more of the following levels, surface, 700, 500, and 300 mb, the day is classified as a "cyclone-day". If the cyclone was present at more than one level only the lowest level was used in the statistics.

Except for a few cases, Worthley determined that fronts and cyclones were mutually exclusive within a radius of about 500 miles of the island chain.
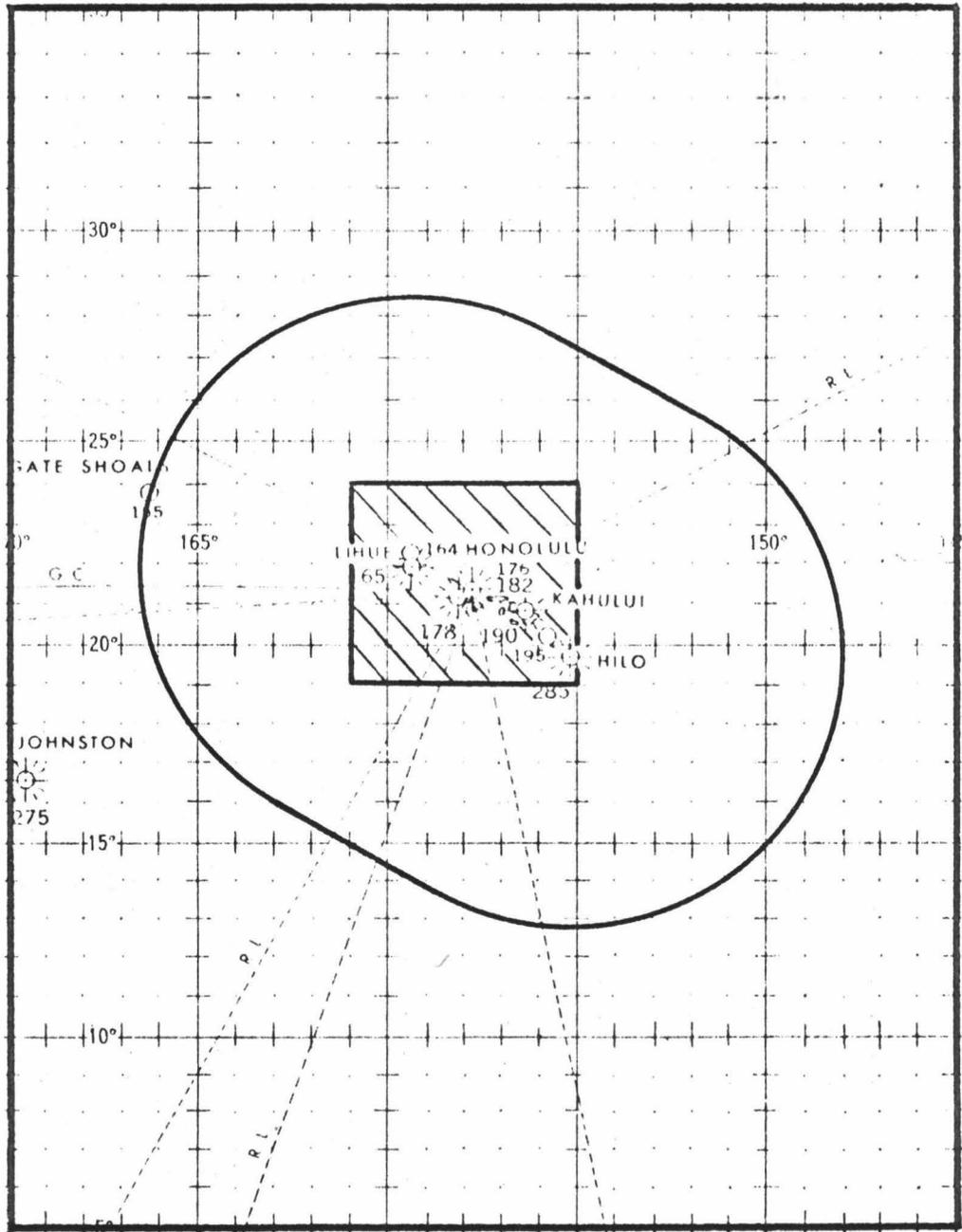
Fig. 4. Areas for fronts (hatched) and cyclone
         (elliptical) tabulations.

Table 2 shows the frequency distribution of thunder-
storm-days with cyclones at various pressure surfaces. Of
a total of 623 cyclone-days thunderstorms or lightning were
reported on only 138 days or about 22 per cent of all cases.
The table shows that thunderstorm-days are most frequent
with cyclones at the surface (61 days out of 146). This
undoubtedly reflects the influence of friction induced low
level convergence. Friction induced low level convergence
may provide sufficient vertical motions to release convec-
tive instability. In addition, the low level convergence
is probably associated with vertical stretching which
increases the lapse rate and increases buoyancy. Thunder-
storm-day frequencies decrease progressively when the
closed circulations are only observed aloft, with the
exception of a relatively high frequency at 300 mb. This
high frequency is due to the small number of cases when
cyclones at 300 mb were not associated with cyclones at lower
levels. This in turn may be due to the lack of data, since
most of the analyses were performed prior to the era of jet
transport.

The table suggests that lows aloft are inefficient
producers of severe convective activity. Although the mid-
tropospheric disturbances may produce convergence and
rising motions above the level of the disturbance, the verti-
cal motion field below is apparently characterized by
subsidence.

No. of Cyclone-days

Month	with a thunderstorm-day				without a thunderstorm-day				Total
	300mb	500mb	700mb	Sfc	300mb	500mb	700mb	Sfc	
Jan	1			5	1	3		9	19
Feb	1	1	5	12		9	1	13	42
Mar	1	2	2	16		9	2	15	47
Apr		3	2	3	1	4	3	6	22
May		6	8	2	1	20	19		56
Jun		4	1	2	3	50	11	3	74
Jul		2	4		2	42	36	2	88
Aug		5				42	22	4	73
Sep		6	2	5	1	30	16	7	67
Oct	3	7	5	8		19	14	7	63
Nov		1		5	6	4	12	11	39
Dec		2	3	3	2	14	1	8	33
Total	6	39	32	61	17	246	137	85	623

Table 2. Frequency of cyclone-days with thunderstorm-days
at Lihue, Honolulu or Hilo Jan. 1950-Sept. 1966.
Blanks indicate no occurrence. Pressure levels
indicate lowest pressure surface at which the
closed circulation existed.

Over the Hawaiian Islands, peak activity with mid-
tropospheric cyclones (but minimum thunderstorm activity)
is observed during the summer months (Worthley, 1967).
During this period the prevailing low level, subsiding
easterly flow from the subtropical ridge north of the
island chain is then overlain by high level westerlies.

MAP TYPES

The final comparison with synoptic-scale patterns was
made using the classification system of Yeh et al (1951) and
modified by Worthley (1967). In this comparison, each
thunderstorm-day during the period is tabulated against the
prevailing map type. The number of years of data is slightly
reduced from the number used in the previous two sections,
due to the non-availability of data. In addition, only
the months of October-March are considered since the bulk
of thunderstorm activity is observed during this period.

Table 3 lists the results of this comparison and Fig. 5
depicts the map type classifications. Table 3 shows that
thunderstorms occurred most frequently with type 5, corres-
ponding to the classical "Kona" patterns. However, signi-
ficant contributions were also made by types 1a and 1b.
While it could be expected that type 5 would be significantly
related with thunderstorms, it is interesting to note the
amount of activity associated with types 1a and 1b, both
normally associated with tradewind (i.e. northeast to east)
surface flow over the islands. The large number of thunder
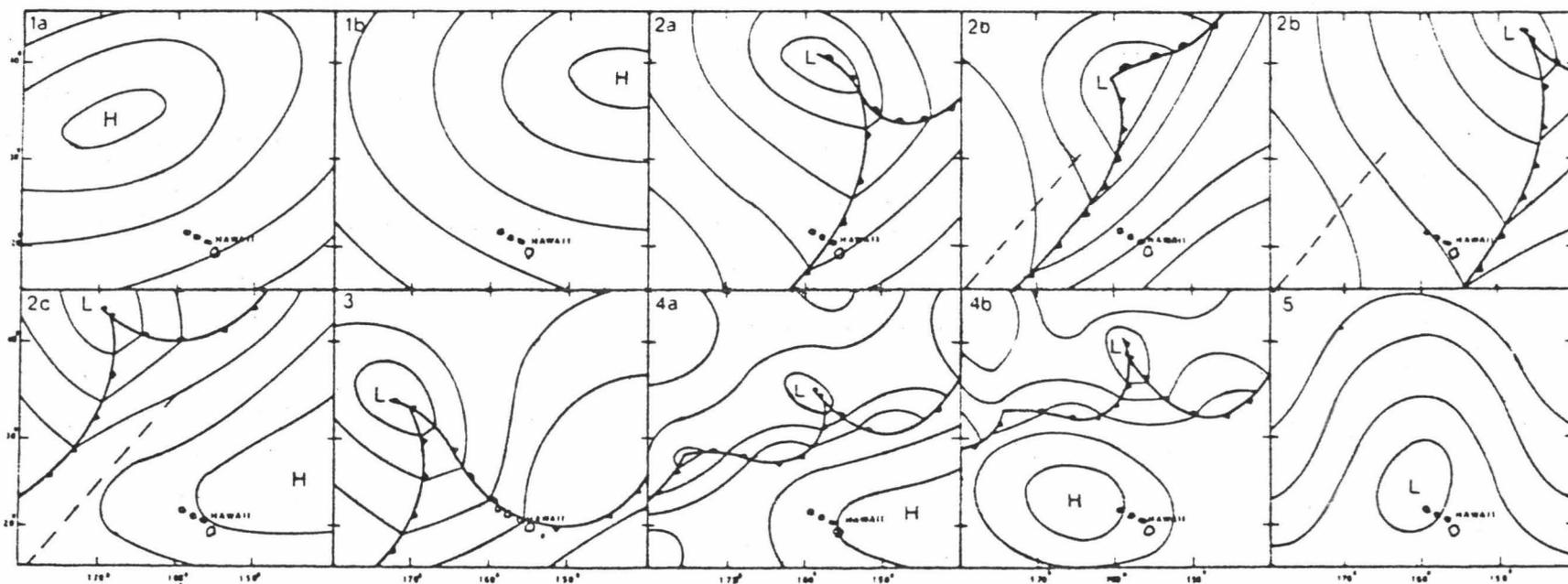
Fig. 5. Map types after Yeh et al. (1951).
(Type 2c added.)

Map Type	1a	1b	2a	2b	2c	3	4a	4b	5	Total
Total No. for period	517	560	166	450	311	6	210	12	161	2393
No. with thunder-storm-day	56	75	25	63	31	2	15	0	85	352
Percent of total with thunderstorm-day	11	13	15	14	10	33	7	0	53	100
Percent of total thunderstorm-days	16	21	7	18	9	1	4	0	24	100

Table 3.   Frequency of thunderstorm-days with map types
(after Yeh et al (1951) and Worthley (1967))
Oct.-Mar. 1950-55, Oct.-Mar. 1958-64, Jan. 1966.

storms associated with the tradewinds may indicate the influence of topographic lifting in generating cumulonimbus, but it is also interesting to note that the percentage of tradewind days with thunderstorms is low. Types 1a and 1b, reflecting the more frequent synoptic patterns over the islands, account for about 45 per cent of all map types. Only 11 per cent of type 1a patterns, however, were accompanied by thunderstorm-days; for type 1b, only 13 per cent. Types 2a and 2b individually and collectively were correlated with thunderstorm-days only slightly more than 14 per cent of the time, while type 2c had a frequency of only seven per cent. Type 5, the category most significantly related to thunderstorms, has a frequency of 53 per cent.

Interestingly, types 1a, 1b, 2b, and 5 have roughly similar frequencies of the <u>total</u> number of thunderstorm-days (16, 21, 18, and 24 per cent, respectively). Collectively, types 1a and 1b accounted for 37 per cent of the <u>total</u> number of thunderstorm-days while the frequency for type 5 was only 24 per cent; the remaining 39 per cent is accounted for by all other types combined.

The table shows that, while thunderstorms occur relatively infrequently with all map types (with the exception of map type 5), nearly four out of ten of <u>all</u> thunderstorm-days take place during the so-called tradewind patterns.

TEMPERATURES

In the course of the investigation, it was observed that thunderstorms occurred more often with cooler than normal temperatures aloft, particularly at 300 mb. This tendency suggested cold advection aloft as a possible initiator of thunderstorms. In order to test the apparent relationship of thunderstorms with temperatures aloft, mean temperatures at 300 mb and 700 mb over Lihue, Kauai for the months October-March were computed and thunderstorm-days at Lihue, Honolulu, or Hilo were tabulated against the occurrence of temperatures higher or lower than the means. The tabulations were made separately for 300 mb and 700 mb and for each month.

Tables 4-6, summarizing the results, show that thunderstorms do occur more frequently with cooler than normal temperatures aloft. The contingency tables indicate statistically significant values of chi-squared. However, it is clear from the tables that, while the probability of thunderstorms is enhanced by cooler than normal temperatures aloft, attempts at temperature correlation would be of little practical significance for forecasting thunderstorms. The problem is rendered more difficult since it is impossible to recognize whether the cooling is in response to advection, vertical motion, or diabatic effects.

Month	$\overline{T}_{700}$	s_{700}	$\overline{T}_{300}$	s_{300}
Jan	6.4	2.5	−36.3	3.6
Feb	5.8	2.8	−35.4	3.9
Mar	4.7	2.2	−38.2	3.3
Oct	7.7	2.2	−34.3	1.9
Nov	7.3	2.5	−36.1	2.1
Dec	6.8	2.4	−36.7	2.8

Table 4. Mean (barred) 1200 GMT 700mb and 300mb temperatures (^{0}C) and standard deviations(s) for Lihue 1958, 1960-63. Subscripts denote pressure levels.

	$\overline{T}_{700}$	$\overline{\overline{T}}_{700}$	Sum
Thunderstorm occurred	95(60)	35(70)	130
No thunderstorm	326(361)	451(416)	777
Sum	421	486	907

chi-squared = 44.0

Table 5. Frequency of thunderstorm-days at Lihue,
Honolulu or Hilo with Lihue mean 700mb
temperature (Table 4) Oct.-Mar. 1958, 1960-63.
No relation values in parantheses.

	$\overline{T}_{300}$	$\overline{\overline{T}}_{300}$	Sum
Thunderstorm occurred	84(62)	45(68)	130
No thunderstorm	350(372)	428(405)	777
Sum	434	473	907

chi-squared = 17.5

Table 6. Frequency of thunderstorm-days at Lihue,
Honolulu or Hilo with Lihue mean 300mb
temperature (Table 4) Oct.-Mar. 1958, 1960-63.
No relation values in parentheses.

TEMPERATURE CHANGES

From the earlier study it was evident that the marked semi-diurnal periodicity of Hawaiian thunderstorms required a systematic investigation of temperature changes and the methods by which they are produced.

The local time rate of change of temperature can be expressed primarily as a function of three major components and is shown by the relationship

$$\frac{\partial T}{\partial t} = \frac{1}{c_p}\frac{dq}{dt} - \bar{V}\cdot\bar{\nabla}T - w(\gamma_d - \gamma)$$

(1)

(Definitions of the various symbols in this
and subsequent equations are contained in
the appendix.)

The first term of (1) represents the contribution due to non-adiabatic effects. The second term is the advection term and the last represents changes due to ascent or descent.

Vertical Motion

Calculations show that modest values of vertical motion can produce significant temperature changes. It can be shown by (1) that an updraft of six cm/s, acting alone, will produce changes in the order of one °C/hr. However, vertical motion is difficult to determine objectively and it is never routinely observed.

Diabatic Effects

Radiative heating and cooling, and latent heat release, are two of the important factors in the diabatic term of (1). Although many texts and articles allude to the efficacy of cloud top cooling in the production of nocturnal thunderstorms, very few contain a quantitative treatment.

In the absence of insolation and of other dynamic or thermodynamic mechanisms long-wave flux will cool the upper part of the cloud (see Fig. 6). Assuming an exponential decay rate with depth, the cooling rate at any level within the cloud can be expressed as

$$\left(\frac{dT}{dt}\right)_x = \left(\frac{dT}{dt}\right)_o e^{-\alpha x} \tag{2}$$

where the depth is measured downward from the top of the cloud. Since the decay is exponential, if a value of one per cent is ascribed to the base of the layer with respect to the cloud top, then

$$1 = 100 e^{-\alpha x}$$

and

$$\left(\frac{dT}{dt}\right)_x = \left(\frac{dT}{dt}\right)_o e^{-10^{-4} x} \tag{3}$$

The order of magnitude of α would remain essentially unchanged if the cooling rate at the bottom of the cloud layer with respect to the top of the cloud is assumed one order of magnitude smaller or larger than one per cent. This can be verified by substitution.
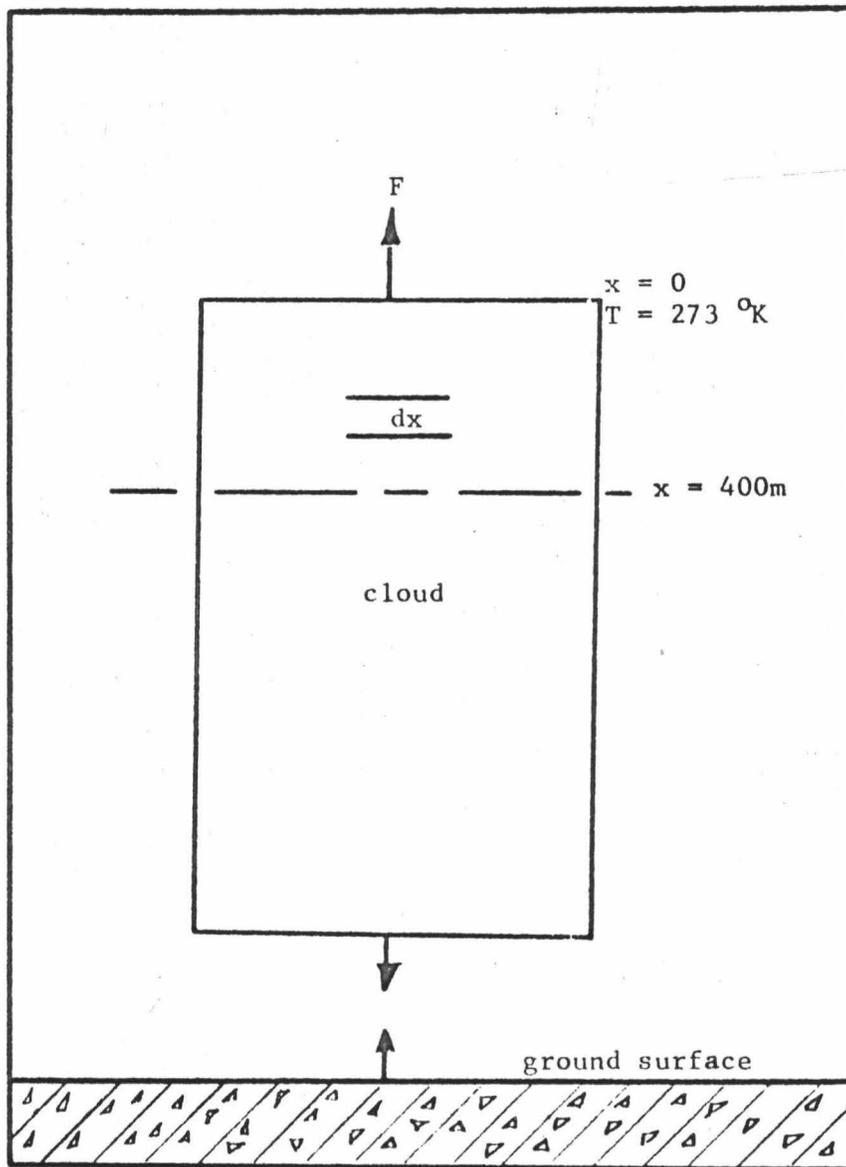
Fig. 6. Schematic of radiative cloud model

Neglecting other sources, the energy balance through the layer can be expressed as a relationship between the radiative flux and the sensible and latent heat losses as a consequence of cooling.

$$\int \rho dx \left[c_p \frac{dT}{dt} + L \frac{d\omega}{dt} \right] = dF \qquad (4)$$

Equation (4) can be shown in the alternate form

$$\int \rho dx \frac{dT}{dt} \left[c_p + L \frac{\epsilon}{\rho} \frac{de}{dT} \right] = dF \qquad (5)$$

The quantity within the brackets is insensitive to small changes and can be evaluated by tables or the Clausius-Clapyron equation and can be shown to be

$$c_p + L \frac{\epsilon}{\rho} \frac{de}{dT} \sim 0.5 \, cal \, g^{-1} \, {}^{\circ}c^{-1} \qquad (6)$$

Thus, (3) and (5) can be combined. Substituting for (6) the relationship becomes

$$0.5 \rho dx \left(\frac{dT}{dt} \right)_o e^{-10^{-4}x} = dF \qquad (7)$$

Since $\left(\frac{dT}{dt} \right)_o \neq f(x)$, integrating from base to top of the layer,

$$\left(\frac{dT}{dt} \right)_o = - \frac{F(10^{-4})}{0.5 \rho} \qquad (8)$$

Tabulations of F derived from the Stefan-Boltzman Law show
that

$$F \sim 0.4 \; cal \, cm^{-2} \, min^{-1}$$

Assuming an average value of $\rho = 10^{-3} g/cm^3$ and that the
ratio of net flux to total flux leaving the cloud top is
one-half due to back radiation and solving (8), then,

$$\left(\frac{dT}{dt}\right)_0 = -2.4 \; ^{\circ}c \, hr^{-1}$$

This amount of cooling is significantly large to be an
important, if not decisive, factor in promoting or maintain-
ing overturning. It is recognized that other considerations
such as convection and long-wave flux through the bottom
and sides of the layer have been neglected.

Similar estimates can be made of warming occurring at
the base of the cloud due to long-wave flux originating
from the ground. Consider a cloud base 1000 feet above
ground with temperature 295 $^{\circ}K$ and surface temperature of
300 $^{\circ}K$. Because of the small temperature difference the
cloud flux differs only slightly from the flux originating
at the ground. The effective net flux impinging on the
cloud base, allowing for absorption in the layer between
ground and cloud, is only about 0.025 $cal/cm^2/min$. A cal-
culation would indicate roughly one order of magnitude
smaller than the value calculated above for cloud top cooling.

During the daytime the effect of insolation, assuming a cloud albedo of 80 per cent and effective incoming short-wave radiation of one langley/min, would completely cancel the upward flux through the tops of the cloud resulting in no cooling at cloud top.

Advection

The horizontal advection of temperature is simple to determine objectively. Utilizing the thermal wind and barometric equations it can be shown that

$$-\overline{V}_g \cdot \overline{\nabla}_p T \;=\; -2\left[\frac{k}{R}\left(\ln\frac{p_1}{p_2}\right)^{-1}\left(\frac{1}{2}V_1 V_2 \sin\theta\right)\right] \tag{9}$$

The relationship shows that the horizontal advection is proportional to twice the area enclosed by the wind vectors.

Table 7 depicts the mean values of the advective tendency for the period 0300 GMT 10 Feb to 1500 GMT 18 Feb 56 computed by (9). The computations were made for 50 mb layers from 1000 to 200 mb. In general, mean advective tendencies are of the order 10^{-2} $^{\circ}$C/hr.

The previous sections have shown that vertical motions and diabatic effects can have orders of magnitude of one $^{\circ}$C/hr. The advective tendency is at least one order of magnitude less than one $^{\circ}$C/hr. The comparison indicates that the actual temperature changes result primarily from two interacting factors. The dominant factor cannot be positively identified at any particular time.

P(mb)	$-(\overline{V}\cdot\overline{\nabla}T)\times10^{-2}\ {}^{o}C/hr$	
	0300 GMT	1500 GMT
1000		
	9.7	20.2
950		
	12.6	21.2
900		
	7.2	17.3
850		
	6.1	19.8
800		
	7.9	-14.8
750		
	9.7	34.6
700		
	15.9	- 9.4
650		
	- 7.6	- 9.4
600		
	- 4.7	0.7
550		
	-10.4	6.5
500		
	6.5	- 6.8
450		
	19.1	-10.4
400		
	3.6	- 3.6
350		
	- 9.0	54.7
300		
	8.3	41.4
250		
	43.6	-10.4
200		

Table 7. Average computed advective temperature tendency. 0300 GMT 10 Feb. - 1500 GMT 18 Feb. 56 for Lihue.

STABILITY

Figures 7-9 represent time sections of virtual-equiva-
lent-potential temperatures over Lihue for three selected
periods. The occurrence of electrical activity at Lihue
during these periods is marked on the lower portions of the
diagrams. The significance of these diagrams is found in
the fact that convective instability is nearly always pre-
sent in the air mass over the Hawaiian Islands. The
convective instability usually results because of the
moisture distribution. Abundant moisture present in the
lower levels normally decreases rapidly with height.

Attempts at correlating electrical activity with the
axes of the minima of virtual-equivalent-potential tempera-
ture, or the maximum height of the convectively unstable
layer, were unsuccessful. Attempts to correlate thunder-
storms with the vertical gradient of virtual-equivalent-
potential temperature or with specific minimum values of
virtual-equivalent-potential temperature were similarly
unsuccessful.

Graphical computations on thermodynamic charts show
that small lifts, usually 100 mb or less, of the layers
below 700 mb are more than sufficient to release the poten-
tial instability. Mechanical lifting of these magnitudes
can easily be realized over the mountain ranges of Kauai and
Oahu, ranging up to 4500 feet above sea level. Why is it
that more thunderstorms do not occur with conditions so
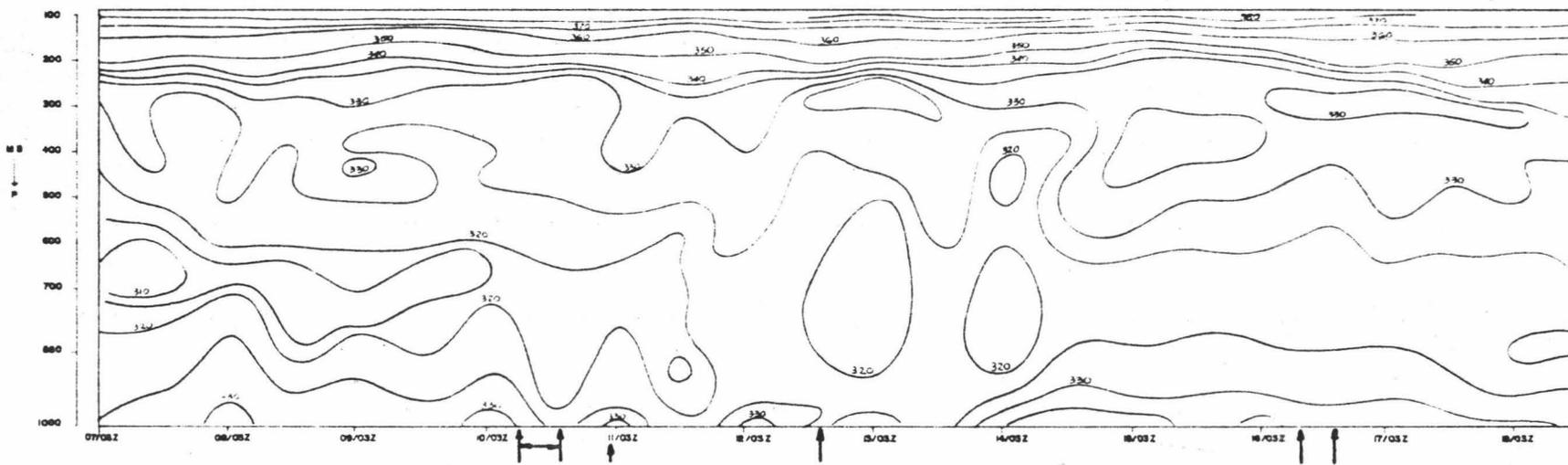
Fig. 7. Virtual-equivalent-potential temperature, °K Lihue Feb. 1956
Thunderstorm occurrences marked by arrows.
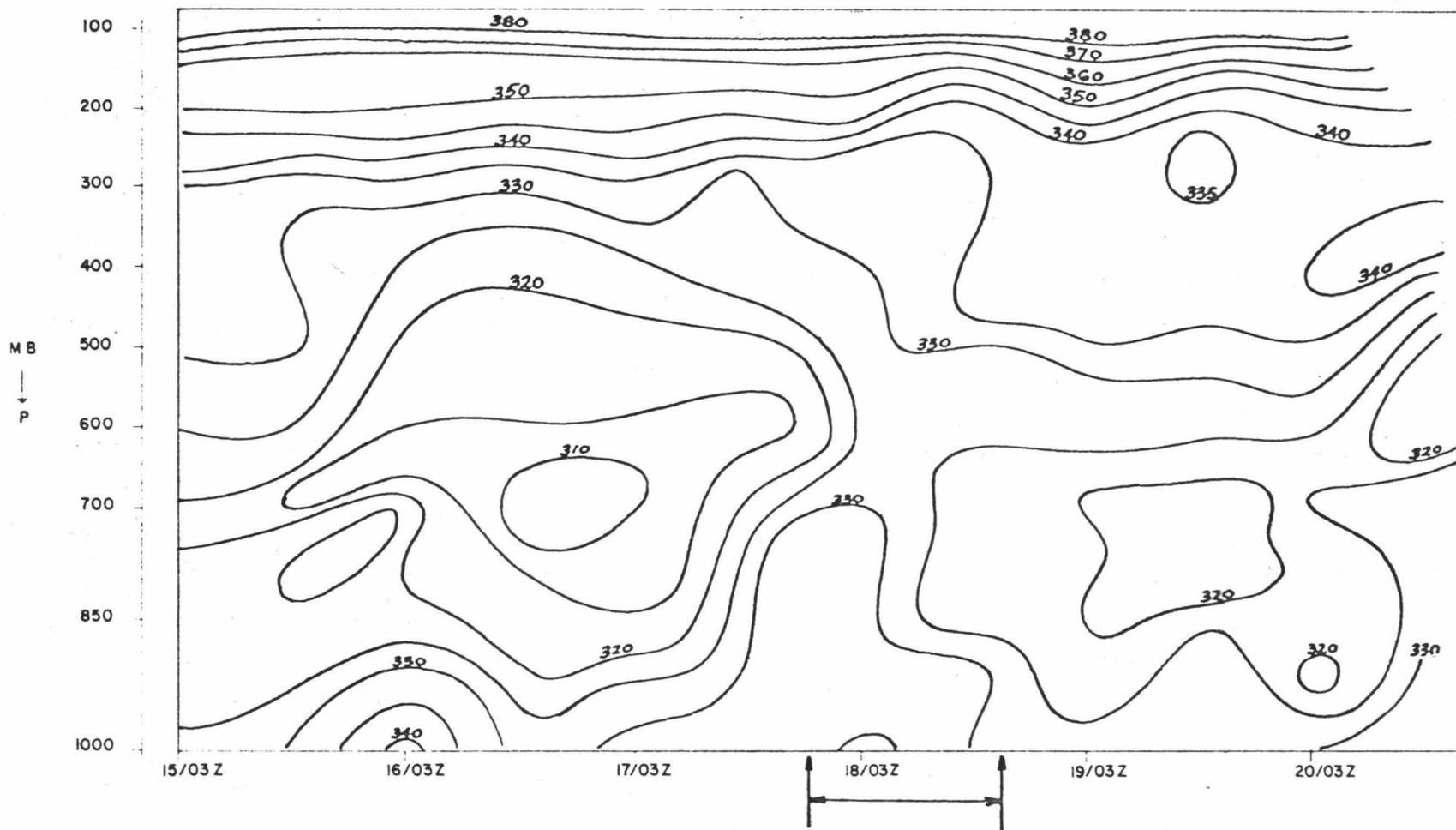
Fig. 8. Virtual-equivalent-potential temperature, °K Lihue Jan. 1957
Thunderstorm occurrences marked by arrows.

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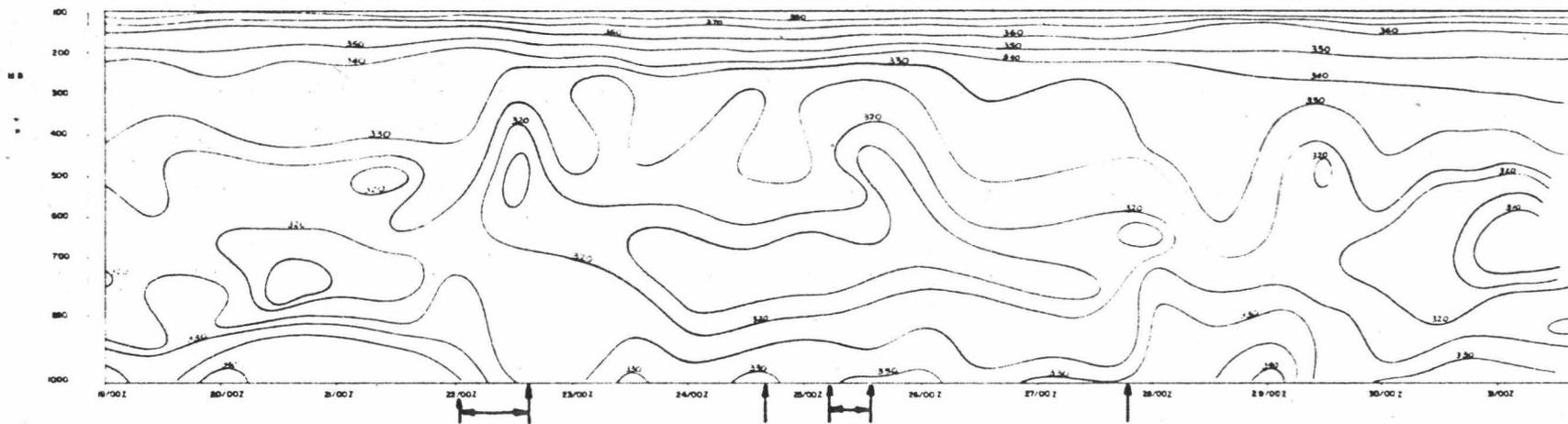
Fig. 9. Virtual-equivalent-potential temperature, $^\circ$K Lihue Jan. 1962
Thunderstorm occurrences marked by arrows.

suitable for their development? The absence of more fre-
quent and severe convective activity demonstrates that the
latent instability is not being released by simple orographic
lift.

RELATIVE HUMIDITY

Figures 10-12 represent time sections of relative
humidity for the same periods shown in Figs. 7-9. Electri-
cal activity at Lihue is shown on the lower portions of
the diagrams. These diagrams show systematic variations
of moisture with periods of two to three days.

Figure 10 shows that prolonged thunderstorm activity
at Lihue commencing 0800 GMT 10 Feb 56 and continuing
through 1700 GMT was preceded by increasingly higher values
of relative humidity (RH) beginning about a day and a half
prior to the onset of thunderstorms.

Similarly, continuous thunderstorm activity at Lihue
began on 2200 GMT 17 Jan 57 and continued through 1800 GMT
18 Jan. Fig. 11 reveals a sharp trend towards increasing
values of relative humidity with time and height commencing
about 2100 GMT 16 Jan or nearly a full day prior to the
onset of thunderstorms.

The same feature can be noted at 0000 GMT 21 Jan 62
(Fig. 12) which preceded the continuous activity at Lihue
commencing 0100 GMT 22 Jan and continuing through 1500 GMT.
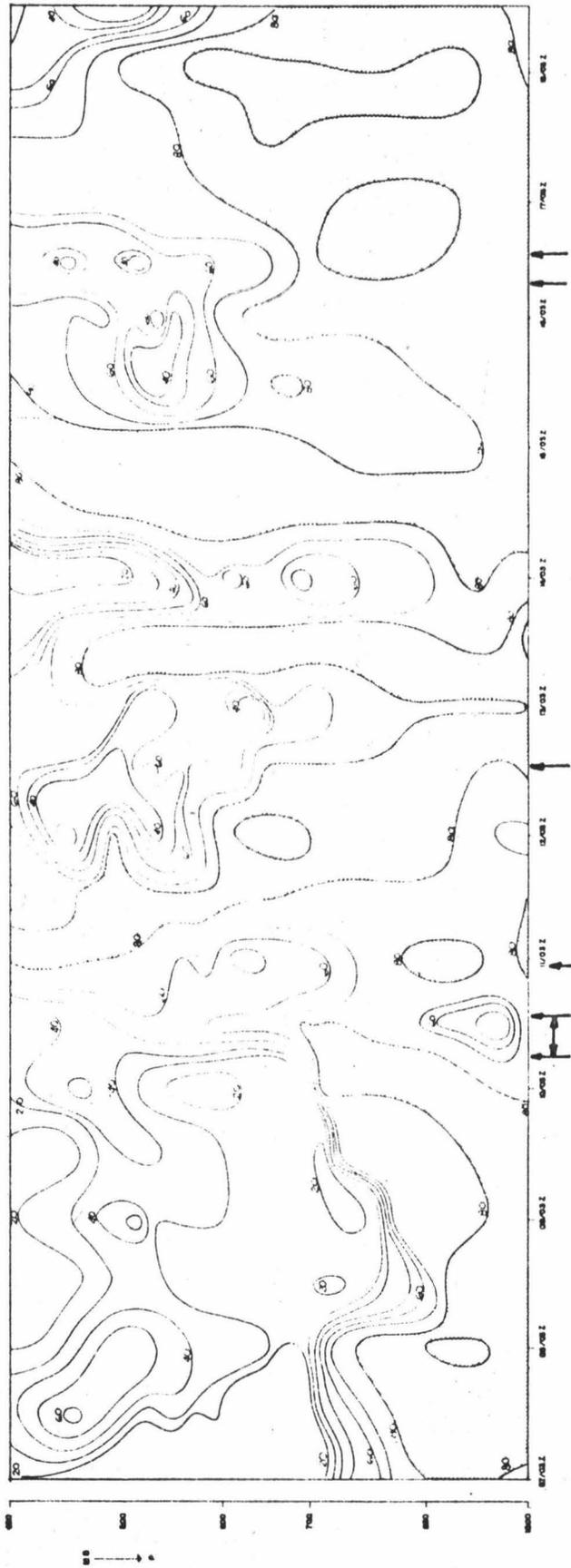
Fig. 10. Relative humidity - % Lihue Feb. 1956
Thunderstorm occurrences marked by arrows.
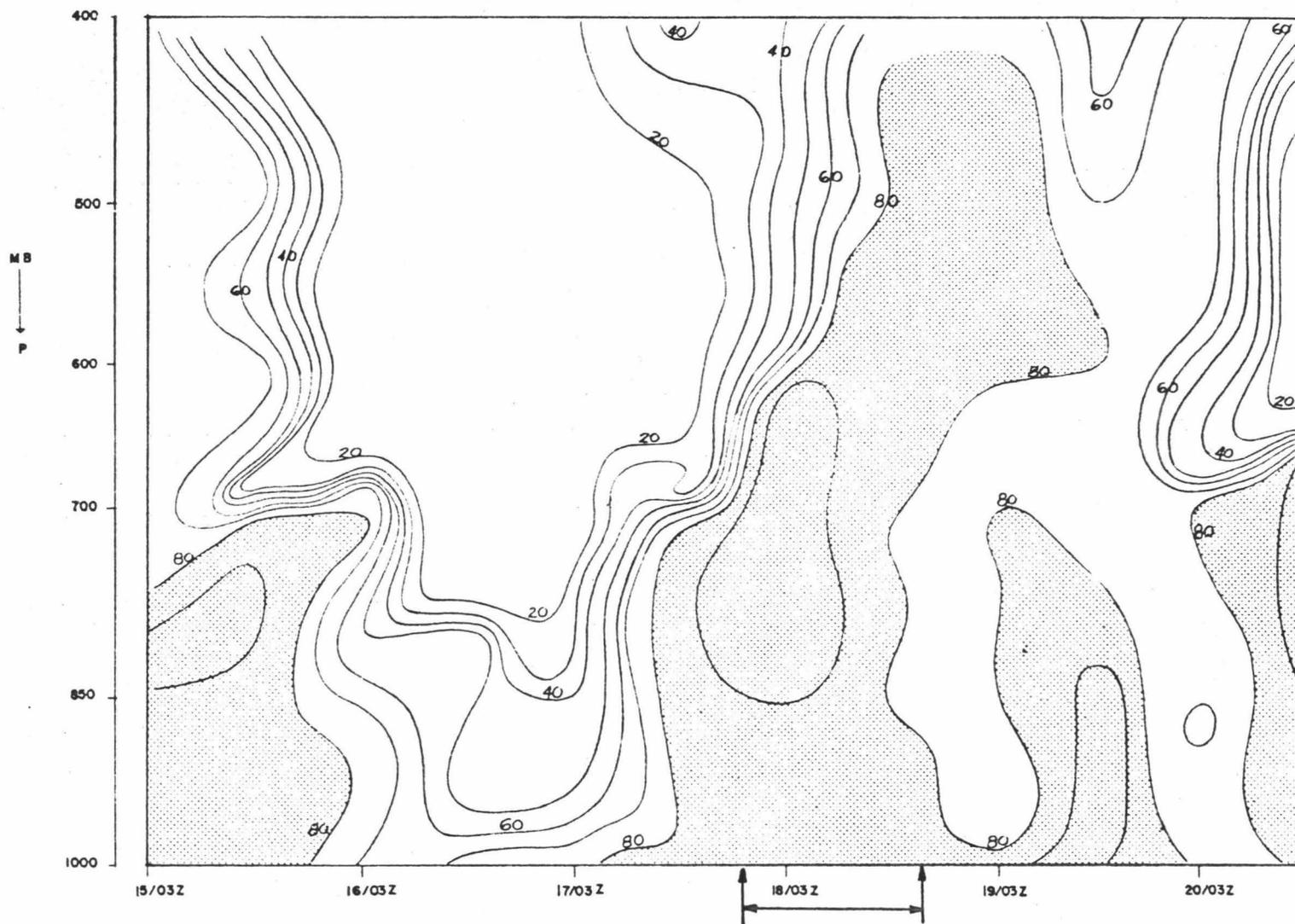
Fig. 11. Relative humidity - % Lihue Jan. 1957
Thunderstorm occurrences marked by arrows.
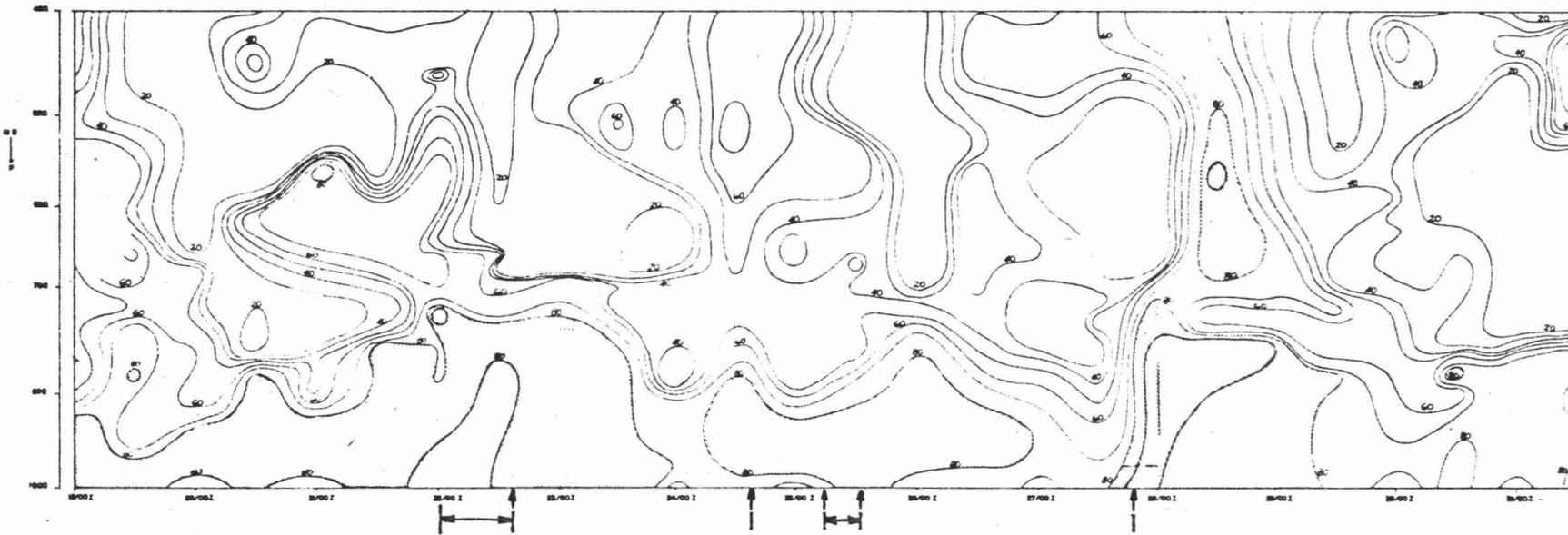
Fig. 12. Relative humidity - % Lihue Jan. 1962
Thunderstorm occurrences marked by arrows.

The patterns were not entirely consistent, however. Increasing values of RH subsequent to 0300 GMT 13 Feb 56 did not culminate in significant thunderstorm development at Lihue despite a deep moist layer extending to over 500 mb twelve hours later.

Relative humidities were usually high during thunderstorm activity. One exception worthy of note is the relative minimum in the low levels centered on 1500 GMT 10 Feb 56. As observed previously, thunderstorms prevailed nearly continuously from 0800-1700 GMT. It could be argued that the low level RH minimum resulted from the dessicating effects produced by downdraft motion as thunderstorms were in progress at the time the sounding was made. However, inspection of the hourly records at Lihue provided no substantiating evidence of gusty downdraft conditions during the period.

In routine forecasting use of auxiliary charts such as RH diagrams is prohibited by the difficulty of objectively determining the trend of the pattern, which is essentially a problem involving a second order prognostic equation. A second difficulty is in recognizing whether the changes represent responses due to horizontal advection of moisture or those due to lifting of moisture from lower levels as a result of vertical motions.

PHYSICAL MODELING

A physical cloud model designed for computer application has been developed by Weinstein (1966). The model simulates steady state cumulus convection.

The three basic equations of the model, energy thermodynamic, and moisture equations, yield steady state profiles of vertical motion, cloud temperature, temperature excess of cloud over environment, cloud mixing ratio, and liquid water content of the cloud. The printed output also yields maximum cloud top, amount and duration of rainfall, and updraft area at various stages of development.

Input to the model consists of a sounding of pressure, temperature, relative humidity, and wind (optional) at all mandatory and significant levels. An assumed cloud base height and updraft radius are additional input parameters.

The entrainment parameter is assumed to be

$$\mu = \frac{.2}{R} \qquad (10)$$

The model is similar, and the entrainment rate identical, to the plume model considered by Squires and Turner (1962). Morton's (1960) values of the entrainment constant with cylindrical profiles has been slightly modified from 0.116 to 0.10 in Weinstein's model.

The model is sufficiently versatile to incorporate many options of development. In addition to those already mentioned the ice nucleation temperature, effects of wind

shear, autoconversion and accretion exchange rates, buoyancy reduction, updraft radius, and initial impulse at cloud base may be initially specified to simulate various conditions.

The ice nucleation temperature is initially specified to simulate spontaneous (248 ^oK) or artificial nucleation (267 ^oK).

The effect of wind shear as a mixing and shearing agent modifying vertical growth can be included as part of a subroutine or it can be omitted.

The autoconversion rate specifies the rate at which cloud water is converted to hydrometeor water and the accretion rate determines the rate at which hydrometeors collect water. Both can be varied to simulate various degrees of efficiency.

Cloud buoyancy, used to determine the vertical velocity, must be reduced by the weight of the liquid water carried aloft. The options provided in the program allow for buoyancy retardation by cloud and hydrometeor water or by cloud water alone.

The updraft radius alternately can be kept constant or allowed to vary according to the mass continuity equation. As the vertical velocity increases/decreases with height, the updraft area decreases/increases.

Figure 13 depicts the basic flow diagram used in the model.

1. The model interpolates the initial input sounding for constant height increments of 200 meters.

2. The standard parcel method, with mixing or entrainment, is numerically simulated by the program. The parcel ascends one height increment and is then mixed with the environment at constant pressure.

3. The cloud and hydrometeor water are computed next. The vertical velocity is then computed as the buoyancy less the drag created by the liquid water carried and mixing with the slower environmental air.

4. Calculations for the next higher height interval are again computed provided that the cloud temperature is greater than the ice nucleation temperature. The ice nucleation temperature is specified initially, corresponding to spontaneous or artificial nucleation. If the cloud temperature is less than that of the ice nucleation temperature, the liquid water is frozen and the latent heat gain is added to the computations performed for the next height increment. This increase in energy is added for one step only; thereafter calculations are performed with respect to ice saturation.

READ: Sounding at significant levels
height increment

Interpolate sounding to levels of height increment

READ: Autoconversion & accretion rates
updraft radius (R)
freezing temperature

Compute mixing parameter = 0.2/R

Initialize cloud base:
cloud temperature = environment temperature
parcel saturated
vertical velocity(w_1) = 2.0(1.0)mps
hydrometeor L.W.C. = cloud L.W.C. = 0

Lift parcel moist adiabatically one height interval

Mix parcel with environment at constant height

Generate cloud and hydrometeor liquid water

Compute vertical velocity(w), precipitation, radar
reflectivity factor, temperature excess,
updraft area

Compute total precip duration

$w < 0$

yes   no

Print height, w, etc.

Print

1st time

yes   yes   no

freezing

Lift parcel ice adiabatically one height interval & add heat of freezing

no

Lift parcel ice adiabatically one height interval

END

Fig. 13. Basic flow diagram of steady state model(Weinstein, 1967)

5. The computations continue stepping for 200 meter
    height intervals until downward vertical motions,
    indicating negative buoyancies, have been computed.

Figures 14-17 show some of the results of the model,
tested with a few of the options. The data used were taken
from selected dates for the same periods shown in Figs. 7-9.
The spontaneous ice nucleation temperature was used in all
runs.

Effective discrimination of the period of activity on
10 Feb 56 from the lack of any activity on the 8th and 9th
is shown in Fig. 14. Computed cloud heights for the 10th
were significantly higher than those computed for the 8th
or 9th for all updraft radii. The largest difference,
about 8 km, occurs for an updraft radius of 1.5 km between
the 8th and 10th. The model was not particularly successful
in discriminating the lack of activity on the 17th from
that of the 10th. Computed cloud heights for the 17th,
although generally lower than for the 10th, were quite
similar. There is, however, a significantly large differ-
ence of about 4 km for an updraft radius of 1.5 km. For
updraft radii less than or exceeding 1.5 km the differences
were less substantial.

Figure 15a shows good correspondence for the prolonged
thunderstorm activity that began on 2200 GMT 17 Jan 57 and
continued through 1800 GMT 18 Jan. The computed cloud
heights from the 1500 GMT 17 Jan sounding do not differ

Fig. 14. Cumulus dynamics model cloud height
vs. updraft radius.
Feb. 8 - 10, 17, 1956.
Thunderstorms not observed - solid lines.
Thunderstorms observed - dashed line
Cloud base = 500m.
Initial impetus(w_1) = 2 m/s
No correction for shear

0300GMT 10 Feb

1500GMT 17 Feb

0300GMT 9 Feb

0300GMT 8 Feb

0300GMT 8 Feb

Cloud height - km
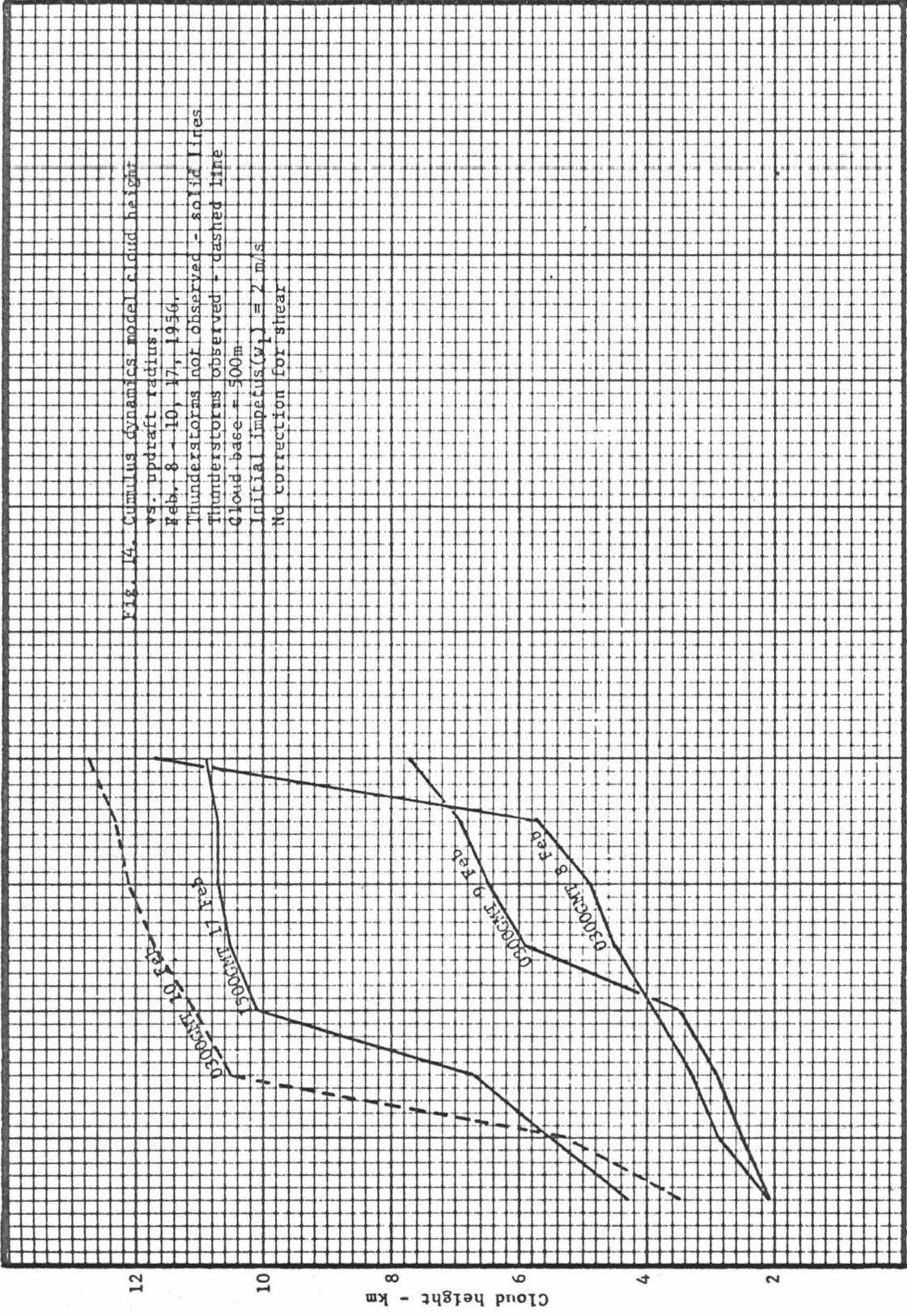
Updraft radius - km

Fig. 15a. Cumulus dynamics model
cloud height vs. updraft
radius.
Jan. 15 - 17, 1957

Thunderstorms not observed - solid
lines
Thunderstorms observed - dashed
line
Cloud-base = 500m
Initial impetus(w_1) = 2 m/s
No correction for shear

Fig. 15b. Same as 15a except
all cases modified
to include shear
correction.

Cloud height - km

Updraft radius - km

1500GMT 15 Jan
1500GMT 16 Jan
1500GMT 17 Jan

Fig. 16. Cumulus dynamics model cloud height
vs. updraft radius,
Jan. 15 - 17, 1957
Thunderstorms not observed - solid lines
Thunderstorms observed - dashed line
Cloud base = 500m
Initial impetus (w_1) = 1 m/s
No correction for shear

1500GMT 17 Jan
1500GMT 16 Jan
1500GMT 15 Jan

Cloud height - km

Updraft radius - km

42

Cloud height - km

12

10

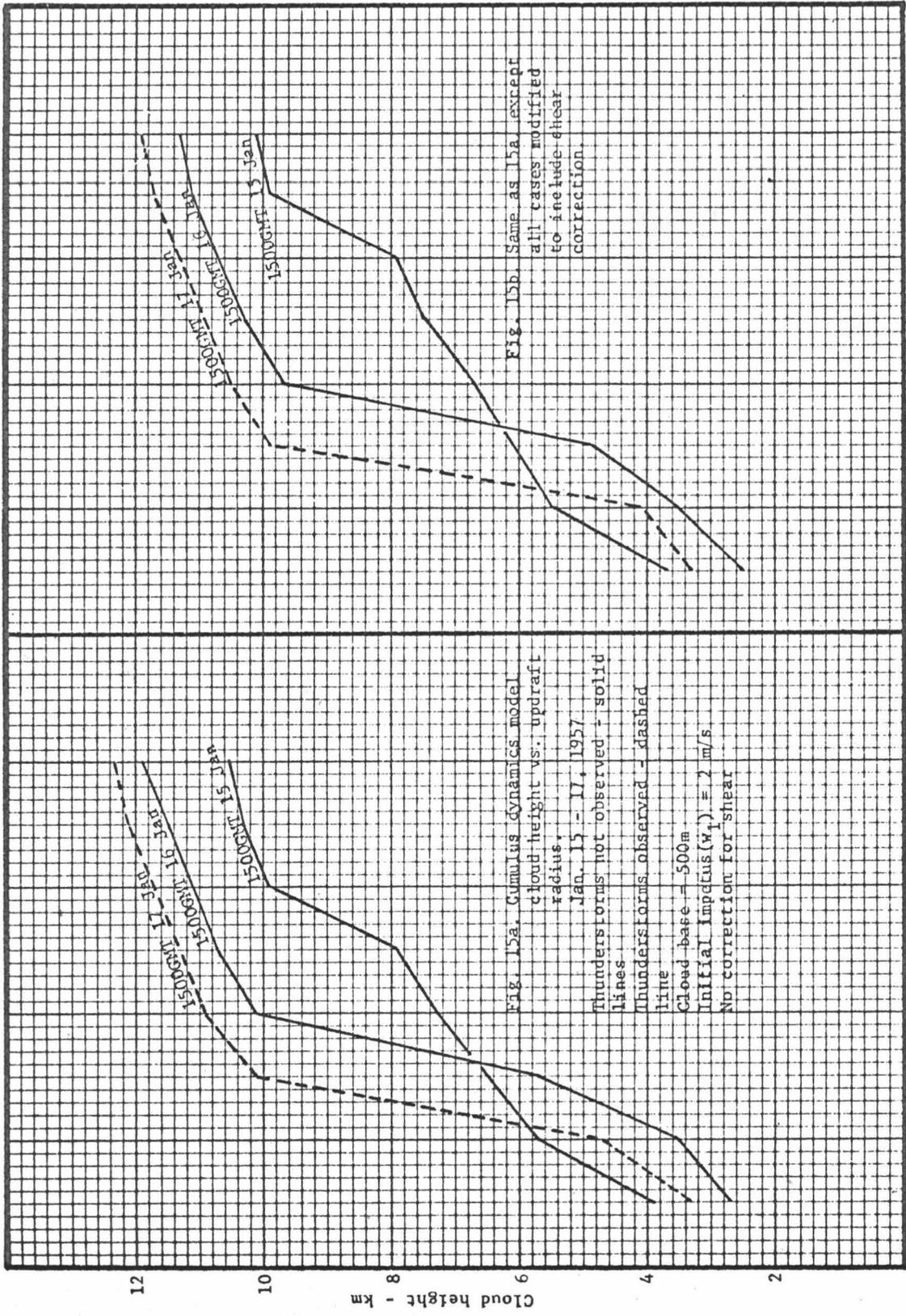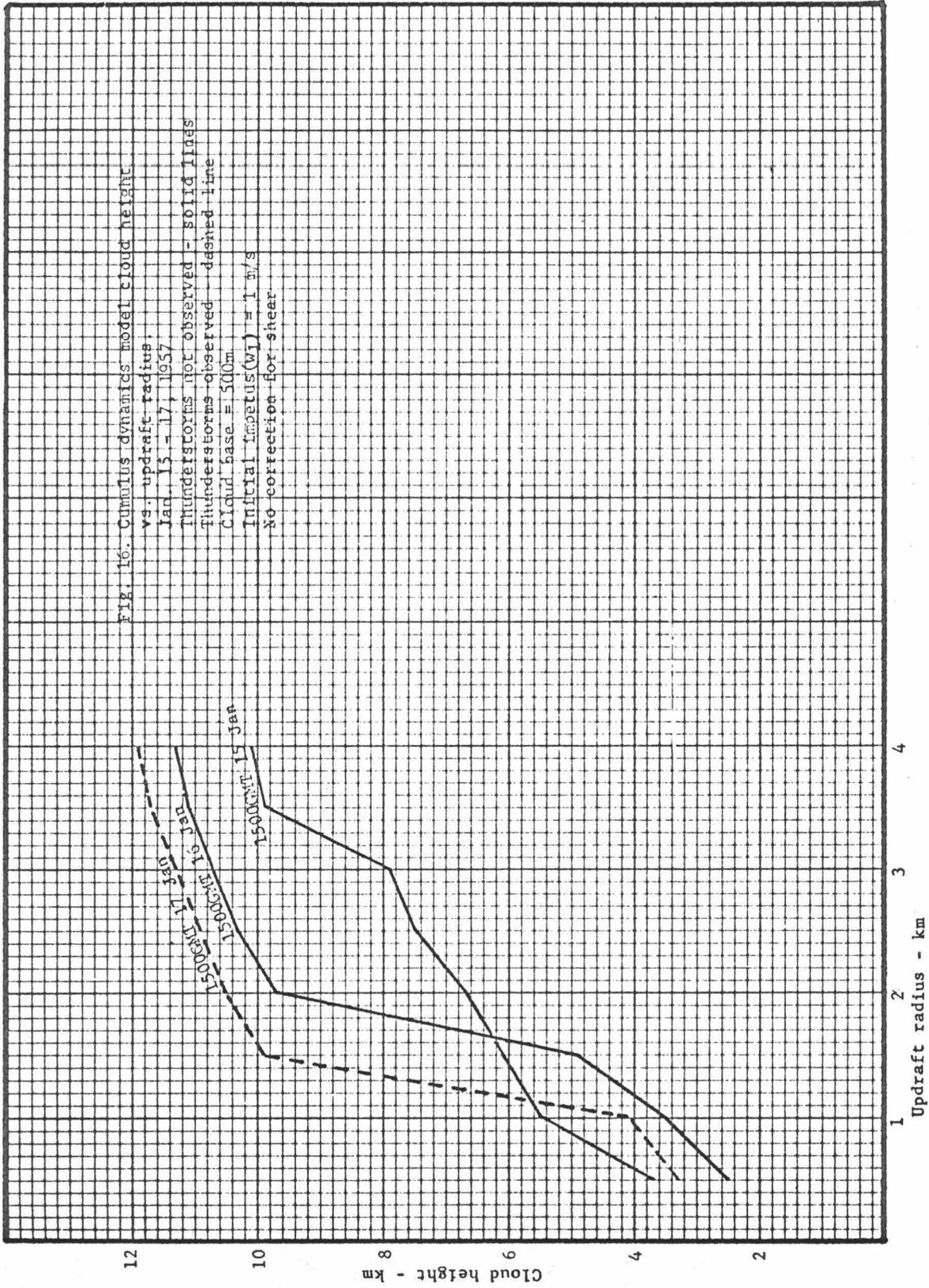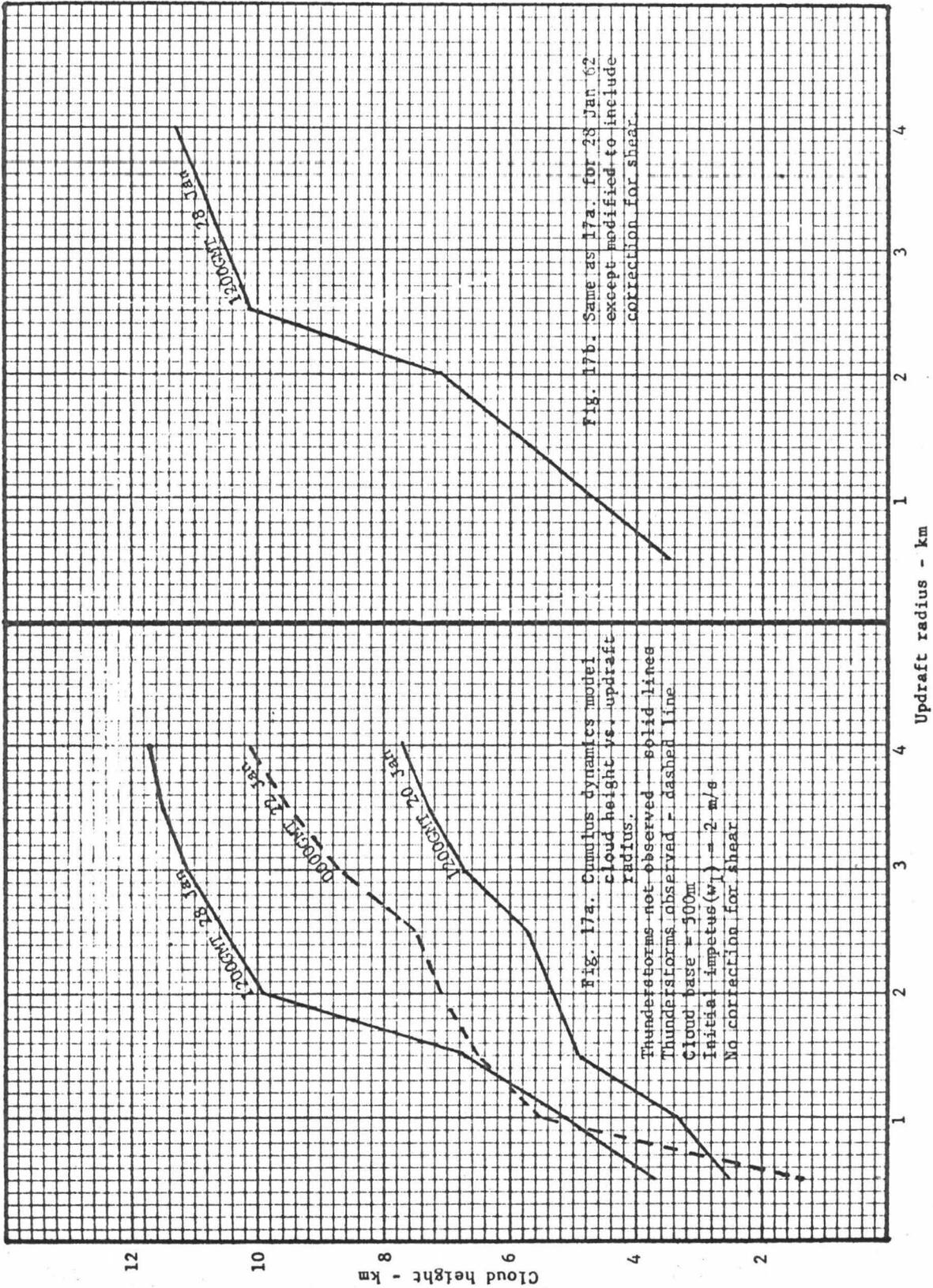8

6

4

2

1200GMT 28 Jan

1200GMT 28 Jan

0000GMT 22 Jan

1200GMT 20 Jan

Fig. 17a. Cumulus dynamics model
cloud height vs. updraft
radius.

Thunderstorms not observed—solid lines
Thunderstorms observed—dashed line
Cloud base = 500m
Initial impetus $(w_1) = 2$ m/s
No correction for shear

Fig. 17b. Same as 17a for 28 Jan 62
except modified to include
correction for shear.

Updraft radius - km

significantly from those computed from the 1500 GMT 16 Jan
sounding, however, except for an updraft radius of 1.5 km.
The difference in the computed cloud heights for an updraft
radius of 1.5 km for the two dates is about 4.5 km. The
model was moderately successful in discriminating the lack
of activity on the 15th, especially for updraft radii from
1.5-3.0 km. However, the results are inconclusive for
radii less than 1.5 km or greater than 3.0 km.

Figure 15b represents the same dates and input para-
meters as Fig. 15a except that the effects of wind shear
have been considered in the computations. This modification
is observed to provide an inhibiting effect. All computed
cloud heights for all dates are lower than those computed
without considerations of wind shear. In spite of large
differences in shear at levels above 300 mb for the three
dates (Tables 8-10), the general configurations are unchanged
from those in Fig. 15a and the modification failed to pro-
vide further discrimination.

Figure 16 again represents the same dates and input
parameters as Fig. 15a. No shear correction has been
included. However, the initial impetus at the cloud base
has been modified from two m/s to one m/s. It is again
observed that the modification tends to decrease the com-
puted cloud heights from those in Fig. 15a. Again, however,
the general configurations and relative placements of the
distributions have not changed significantly. The

P(mb)	T(^{o}C)	RH(%)	V(mps)
950	18.7	82	180/06
850	11.7	85	240/09
795	9.7	91	240/09
707	3.5	74	240/10
700	3.3	58	270/11
682	2.5	22	270/10
668	1.7	54	270/10
618	- 2.7	82	270/10
600	- 3.2	66	270/10
570	- 6.5	59	270/10
500	-11.2	72	270/09
472	-13.5	69	270/10
418	-20.5	28	270/15
400	-22.0	18	270/22
300	-37.1	20*	280/47
250	-41.1	20*	280/50
200	-52.1	20*	280/40
150	-64.9	10*	280/40
100	-75.0	10*	280/30

* assumed values

Table 8. Input parameters for cumulus
dynamics model calculations
1500 GMT 15 Jan. 57.

45

P(mb)	T($^{\circ}$C)	RH(%)	V(mps)
950	20.0	48	260/06
932	19.0	42	260/08
850	13.5	52	260/06
800	10.0	52	260/08
772	9.0	22	260/09
700	6.7	13	240/10
500	-11.7	16	230/09
400	-25.3	19	260/10
300	-38.3	22	260/15
250	-42.9	20*	260/15
200	-53.8	20*	260/15
150	-64.0	10*	260/15
100	-74.2	10*	260/15

* assumed values

Table 9. Input parameters for cumulus dynamics
          model calculations 1500 GMT 16 Jan. 57.

P(mb)	T($^{\circ}$C)	RH(%)	V(mps)
950	19.6	82	200/07
850	13.7	86	180/07
765	5.4	80	190/08
700	5.7	50	190/10
685	5.5	22	190/09
675	4.5	32	190/09
655	3.5	20	190/09
500	-12.7	16	240/20
400	-24.9	48	260/24
300	-39.8	22	260/32
250	-44.9	20*	260/30
200	-54.2	20*	260/30
150	-64.0	10*	260/30
100	-70.7	10*	260/30

* assumed values

Table 10. Input parameters for cumulus
dynamics model calculations
1500 GMT 17 Jan. 57.

modification was not successful in producing further
discrimination.

The results shown in Fig. 17a indicate the model was
particularly unsuccessful in discriminating the activity on
22 Jan 62 from the lack of any activity on the 28th. This
is also true, though less conclusively, for the period of
inactivity on the 20th.

Figure 17b represents the computed cloud heights for
the 28th modified to include the effects of shear. Except
for lowering the individual cloud heights slightly, the
modification produced no significant changes in the results.

The results of these first tests of the model, seen in
Figs. 14-17, are generally inconsistent. While the model
verified the most severe occurrences, it over predicted
cloud heights for those dates on which no thunderstorms
occurred. The model appears to discriminate most effec-
tively for updraft radii of 1.5 km. Interestingly, while
Byers (1965) and Byers and Braham (1949) observed in their
thunderstorm project that the radar cloud associated with
a single thunderstorm cell was about six to ten km in
diameter, aircraft measurements showed the modal width of
updraft and downdraft area to be approximately 1.5 km.

In summary, Figs. 14-17 show the model was able to
separate the most extreme cases. However, the model is
ineffective in discriminating between nearly identical
thunderstorm and non-thunderstorm soundings. In addition,

such as radar reports, satellite photos, observations, etc.,
indicate prior existence of thunderstorms.

Regarding the observed semi-diurnal frequency of
thunderstorms, no evidence has been presented to support
low-level warm advection as a mechanism to explain the
nocturnal thunderstorm maxima in the Hawaiian Islands. Such
a mechanism had been proposed by Means (1944) to explain the
nocturnal thunderstorm maximum over the mid-western United
States. Similarly, no evidence has been presented that
large-scale disturbances maintain periodic tendencies corres-
ponding to the observed semi-diurnal thunderstorm frequencies.
On the other hand, insolation and cloud radiation, because
of their periodic character, can at least qualitatively
support the observed frequencies. Cloud radiation is a
prominent factor to be considered in future work but the
difficulty of obtaining precise and timely measurements
likely preclude much study of this important area.

Results of the study have shown that radiative transfer
and vertical motions are dominant energy processes controlling
convective activity in the Hawaiian Islands. Vertical
motion is apparently the most effective destabilizing
influence.

The study has shown that, although the air mass over
the Hawaiian Islands is nearly always convectively unstable,
the convective instability is inefficiently released and
the triggering mechanism for release is only poorly under-

stood. The fact that many thunderstorms occur during ostensibly quiescent periods (i.e., tradewind patterns) may partially be explained by topographic lifting releasing the ever present convective instability. However, the lack of more frequent convective storms suggest that mechanical lifting over the mountains is often suppressed by strong subsiding motions above the mountains. With strong flow up and over the mountains, strong horizontal divergence can be expected on the windward side such that ascending motion reverses with height to descending motions at elevations above the mountain tops.

The purpose of this study has been partially attained. It has shown that the convective instability is sometimes released by frontally induced low level convergence. At other times, the release of convective instability is achieved by friction induced low level convergence or differential advection associated with cyclonic circulations. At still other times, the convective instability is released by topographic lifting. In addition, results of the cloud model have shown that entrainment is a necessary but not sufficient condition to explain the relatively infrequent occurrence of thunderstorms. Sufficient groundwork has now been established to provide more penetrating attacks on the problem with the prospect of gaining fruitful and positive results.

RECOMMENDATIONS

It is recommended that an additional study be made of observations from cooperative weather observers and military weather units. Such a study should reveal an increase in the number of thunderstorm-days. The study should be used to modify the results presented in this paper.

It is further recommended that the Penn State cumulus dynamics model be tested on a daily operational basis. The model has shown some promise but completely realistic results cannot be expected from the model, mainly because all tests have considered only a static environment. More experience and future modifications of the program, including the effects of a non-stationary environment, may yield more realistic and productive results. While serving as another forecast tool, additional statistics may be helpful in modifying the program. It is aslo recommended that a study be made of additional parameters, such as subsidence over the mountains, and the results used to modify the numerical model.

Table 11. Input parameters for cumulus
dynamics model calculations
0300 GMT 8 Feb. 56.

P(mb)	T(°C)	RH(%)
950	17.3	80
900	14.0	83
850	11.8	79
800	9.7	66
750	7.1	59
722	5.7	71
700	5.5	13
690	4.0	20
658	1.0	21
650	1.6	14
643	0.5	39
600	-2.7	14
550	-7.2	15
500	-12.4	16
450	-18.5	17
438	-20.0	28
400	-25.5	19
350	-33.8	29
300	-43.0	20*
250	-51.7	20*
200	-51.8	20*
175	-55.1	10*

Table 11 (cont.)

P(mb)	T(°C)	RH(%)
150	-63.2	10*
125	-67.6	10*
100	-75.3	10*

* assumed values

Table 12. Input parameters for cumulus
dynamics model calculations
0300 GMT 9 Feb. 56.

P(mb)	T(^{o}C)	RH(%)
950	18.4	82
900	15.8	78
850	13.1	73
800	10.3	68
783	11.0	25
760	10.5	19
750	9.8	19
700	5.9	24
650	1.9	27
600	-2.4	30
550	-7.1	31
530	-9.0	31
515	-10.3	51
500	-11.3	49
450	-15.9	30
435	-17.2	26
400	-23.1	18
350	-30.5	20
300	-38.7	22
250	-48.4	20*
200	-57.6	20*
175	-61.7	10*

Table 12 (cont.)

P(mb)	T(^{o}C)	RH(%)
150	-66.2	10*
125	-71.2	10*
100	-75.0	10*

* assumed values

Table 13.   Input parameters for cumulus
dynamics model calculations
0300 GMT 10 Feb. 56.

P(mb)	T(°C)	RH(%)
950	18.8	80
900	15.2	83
850	12.2	82
800	9.3	80
750	5.7	81
715	3.0	82
700	4.4	32
650	0.7	26
600	-3.4	14
550	-7.8	15
530	-9.0	24
500	-13.9	38
458	-19.0	53
450	-20.0	50
400	-26.9	32
350	-33.2	50
335	-34.7	18
300	-41.7	20*
250	-48.9	20*
200	-56.6	20*
175	-61.3	20*

Table 13 (cont.)

P(mb)	T(^{o}C)	RH(%)
150	-66.1	20*
125	-70.1	10*
100	-72.6	10*

* assumed values

Table 14. Input parameters for cumulus
dynamics model calculations
1500 GMT 17 Feb. 56.

P(mb)	T($^{\circ}$C)	RH(%)
950	18.4	84
900	15.1	86
850	11.6	88
800	9.0	89
750	6.6	88
700	3.4	89
650	-0.9	92
638	-1.3	91
613	-1.7	87
600	-2.9	84
550	-7.0	79
500	-11.6	74
450	-16.8	70
400	-23.5	70
360	-29.9	41
350	-31.1	40
304	-38.5	36
300	-39.9	22
250	-41.1	20*
200	-50.0	20*
175	-56.1	10*

Table 14 (cont.)

P(mb)	T(^{o}C)	RH(%)
150	-63.0	10*
125	-69.6	10*
100	-77.0	10*

* assumed values

Table 15. Input parameters for cumulus
           dynamics model calculations
           1200 GMT 20 Jan. 62.

P(mb)	T(°C)	RH(%)
950	19.0	82
900	15.3	83
850	11.6	84
820	9.3	80
810	8.1	66
792	8.1	22
778	12.0	17
717	7.2	23
700	6.1	26
607	-4.5	69
588	-2.7	22
547	-3.0	15
500	-9.3	15
479	-12.8	25
447	-16.8	53
408	-19.8	25
400	-19.9	17
300	-36.0	21
250	-46.3	20*
200	-57.0	20*

Table 15 (cont.)

P(mb)	T(^{o}C)	RH(%)
150	-66.4	10*
100	-76.3	10*

* assumed values

Table 16. Input parameters for cumulus
dynamics model calculations
0000 GMT 22 Jan. 62.

P(mb)	T(^{o}C)	RH(%)
950	17.7	76
850	13.2	82
755	6.0	87
732	6.0	64
700	4.7	69
690	3.5	55
670	1.5	24
660	0.0	53
610	-4.0	23
500	-13.3	50
400	-23.2	40
300	-37.6	67
250	-47.0	20*
200	-59.0	20*
150	-65.1	10*
100	-72.7	10*

* assumed values

P(mb)	T(°C)	RH(%)	V(mps)
950	19.3	84	230/14
900	16.0	87	230/15
850	12.8	89	240/17
780	8.3	95	240/18
752	4.0	63	240/19
745	4.3	48	240/20
721	5.4	69	240/21
700	4.4	74	240/22
562	-4.8	94	260/26
500	-10.5	81	270/31
432	-17.7	70	280/30
400	-22.5	66	280/29
358	-28.5	63	300/40
300	-36.4	46	320/51
271	-40.0	36	320/51
250	-44.4	20*	310/52
200	-54.4	20*	310/48
150	-66.0	10*	300/50
100	-72.5	10*	290/36

* assumed values

Table 17. Input parameters for cumulus
dynamics model calculations
1200 GMT 28 Jan. 62.

APPENDIX

List of Symbols

α	proportionality constant
c_p	specific heat of humid air at constant pressure
$\overline{\nabla}_p$	del operator in a p-system
ϵ	0.622
e	base of natural logarithms; saturation vapor pressure
F	black-body radiation
f	Coriolis parameter
γ_d	dry adiabatic lapse rate
γ	lapse rate
L	Latent heat of evaporation
μ	entrainment parameter
q	heat per unit mass
p	pressure
p_1	pressure at bottom of layer
p_2	pressure at top of layer
R	gas constant for dry air
ρ	density
θ	included angle between V_1 and V_2
T	temperature
t	time
$\overline{V}_g$	geostrophic mean vector wind
V_1	absolute value of wind at p_1
V_2	absolute value of wind at p_2
ω	mixing ratio
w	vertical velocity

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